Civil Engineering and Applied Solutions

Research Article

journal homepage: ceas.journals.umz.ac.ir

Data-Driven Modeling of Concrete Fracture Energy Using Linear Genetic Programming

Ali Nazari [a], Shahin Lale Arefi [b*]

[a] *Department of Civil Engineering, Sharif University of Technology, Tehran, Iran*
[b] *Department of Civil Engineering, Esfarayen University of Technology, Esfarayen, Iran*

ARTICLE INFO

Keywords:
Concrete fracture energy
Linear genetic programming
Prediction model
Soft computing
Compressive strength
Artificial intelligence

Article history:
Received 5 May 2025
Accepted 16 May 2025
Available online 17 June 2025

ABSTRACT

The fracture energy of the concrete is an important parameter that can be used to identify the fracture process of concrete members, especially when subjected to tension and flexural loading. Practices to measure this property in experiments can be expensive and time-consuming. In this study, a statistical model using Linear Genetic Programming is introduced to predict concretes' fracture energy with three readily measured input parameters, namely, compressive strength, maximum aggregate size, and water-to-cemented ratio. The model was developed and trained based on a dataset of 64 measured experimental values taken from published research. The performance of the model was evaluated using statistical indices such as the coefficient of determination, root mean squared error, and mean absolute error, and compared with previously proposed empirical models. The experimental results show that the proposed LGP-based model is superior to old regression-based equations in accuracy and generalization. This model can be a useful methodology for engineers in design and analysis, minimizing the need for a large amount of laboratory testing.

1. Introduction

Concrete is the most widely used construction material because of its economical, high compressive strength as well as availability of material [1]. Despite its good properties, concrete is brittle in tension and flexural conditions and can fail without prior warning [2]. In this sense, fracture energy (GF) has become a pivotal parameter for the design of concrete structures according to fracture mechanics [3].

Fracture energy is often found using experimental tests such as three-point bending tests, which are time-consuming and expensive, and require specific equipment [4]. Also, large differences in experimental configurations and material properties frequently result in conflicting findings. Therefore, there is a growing interest in developing practical and reliable predictive models for predicting concrete fracture energy by using the fundamental material properties. Fracture energy is the energy per unit area of the crack surface that is dissipated during crack propagation in a quasi-brittle material like concrete [5]. Unlike purely brittle materials, concrete exhibits a softening behavior after peak load due to the development of microcracks and the formation of a fracture process zone [2, 5]. Therefore, the fracture energy also is an important index to quantitatively measure the material's strength against crack propagation. Multiple approaches have been proposed to quantify fracture energy, with the most adopted one being the three-point bending test conducted on notched beams [2, 4]. The total fracture energy can be estimated from the load-displacement curve and the geometry of the specimen[6]. While this method provides reliable results, it requires precise instrumentation and testing conditions, which limit its routine application in practice.

To overcome the experimental limitations, various empirical models have been proposed for estimating GF based on measurable concrete parameters. Bazant and Becq-Giraudon [7] presented a regression-based formula incorporating compressive strength,

* Corresponding author.
E-mail addresses: sh.arefi@esfarayen.ac.ir (S. Lale Arefi).

https://doi.org/10.22080/ceas.2025.29159.1008
ISSN: 3092-7749/
How to cite this article: Nazari, A., Lale Arefi, S. Data-driven modeling of concrete fracture energy using Linear Genetic Programming. Civil Engineering and Applied Solutions. 2025;1(1): 89-99. doi: 10.22080/ceas.2025.29159.1008.

aggregate size, and water-to-cement ratio. Similarly, CEB-FIP (1990) [8] and JSCE (2007) [9] provided standardized relationships derived from experimental studies. Although these models provide basic insights, they often fail to generalize across a wide range of material compositions and conditions. They are usually constrained by the limitations of linear regression and rely on simplifying assumptions [10].

Previous studies have proposed various empirical formulas, primarily relying on regression techniques and simplified assumptions [11–14]. These models often have low prediction accuracy and generalization, especially when other unique datasets are used. To address these limitations, soft computing approaches, including Artificial Neural Networks (ANNs) [15], Support Vector Machines (SVMs) [16], and Genetic Programming (GP) [17], have been gaining importance during the last few years. Of these, Linear Genetic Programming (LGP), which is a variation of GP, has been found to be successful in producing interpretable mathematical expressions with efficient computational performance. Recent studies have demonstrated the success of LGP in modeling soil behavior [18], predicting compressive strength [19], and estimating the uplift capacity of suction caissons [20].

This study aims to develop a robust predictive model for estimating the fracture energy of concrete using LGP. The model utilizes a dataset of 64 experimental records with three input variables: compressive strength of concrete (f'c), maximum aggregate size (dmax), and water-to-cement ratio (w/c). The performance of the developed model is compared with widely used empirical models, including those proposed by Bazant & Becq-Giraudon (2002) [7], CEB-FIP (1990) [8], and JSCE (2007) [9], to highlight the superiority of the LGP approach.

2. Methodology

2.1. Overview of linear genetic programming (LGP)

The evolutionary algorithm (EA) is a branch of evolutionary computation that optimizes solutions by bio-inspired mechanisms with respect to some desired outcome [21]. The unique features of these algorithms make them very appealing: they do not require an exhaustive specification of the problem; they work with scant information, are free from the constraints of a particular fitness function, and support multi-objective optimization simultaneously [21, 22]. These characteristics are what have given evolutionary algorithms a far greater following in comparison with other methods in the area of evolutionary computation.

By looking at the population representation and evolutionary operators used, the evolutionary algorithms are generally classified into four primary categories: Genetic Algorithms (GA), Evolutionary Programming (EP), Evolution Strategies (ES), and GP [23]. The idea of applying genetic algorithms to tree-based encoding was first introduced by John R. Koza [24] in 1994, thereby creating the groundwork for tree-based GP. Due to its complexity and time-consuming nature, early applications of GP were limited to solving relatively simple problems [20]. However, with recent advances in GP methodologies and the rapid growth of computational power, this approach is now applied across a wide range of engineering disciplines.

Linear Genetic Programming (LGP) is a subset of traditional tree-based genetic programming. Unlike tree-based GP, which follows a functional programming style, LGP generates programs in an imperative programming style [19]. In LGP, programs are composed of instructions operating on memory registers or constants, and each instruction transfers the result to a destination register [25]. An LGP program can be viewed as a data flow graph. Unlike tree-based GP, where data flow is constrained by the tree structure, the flexible graph structure of LGP allows for the reuse of sub-program outputs during computation [26]. This results in more compact linear solutions and enables the expression of complex operations using fewer instructions. The extent to which these advantages impact performance depends significantly on the design of effective variation operators [27]. LGP follows machine-level instruction modeling to evolve computer programs capable of predicting target outputs from input-output data. The typical steps in an LGP algorithm are as follows [28]:

Initialization: Generate an initial population of programs randomly.

Competition: Randomly select four programs from the population. Based on their accuracy (fitness), classify two as winners and two as losers.

Reproduction and Variation: Apply crossover by swapping segments between the winner programs to create two offspring. Independently apply mutation to each winner to generate new variants.

Replacement: Replace the losing programs with the newly created offspring. The winners remain unchanged.

Iteration: Repeat steps 2 to 4 until convergence. The algorithm's output is the evolved program that best models the desired system behavior.

In LGP, a single solution population can be divided into several subpopulations, and migration between them facilitates program and population evolution. These subpopulations, or demes, evolve more rapidly than an equally sized panmictic population [9, 27, 28].

2.2. Dataset

Modeling via artificial intelligence methods and Linear Genetic Programming (LGP) relies on datasets derived from engineering experiments or observations. Therefore, the first essential step toward successful modeling is collecting a sufficient and reliable dataset. In this study, to develop LGP-based models, a dataset consisting of 64 experimental records [7, 29] has been utilized. Table

1 presents the experimental data used.

Table 1. Experimental Data for Fracture Energy Modeling.

No.	f'c (MPa)	d_{max} (mm)	W/C	G_F (N/m)	No.	f'c (MPa)	d_{max} (mm)	W/C	G_F (N/m)
1	35	2	0.54	49.3	33	55.55	12.5	0.4	100
2	29	10	0.6	70.5	34	58.6	12.5	0.36	100
3	58.9	10	0.4	80.8	35	89	10	0.29	180
4	33.1	10	0.55	76.6	36	56	20	0.45	165
5	44.3	2	0.5	61.3	37	88	10	0.32	140
6	42.1	5	0.5	61.4	38	60.5	20	0.48	120
7	39.9	10	0.5	65.8	39	48	9.5	0.34	116
8	37.6	20	0.5	93.9	40	49	9.5	0.3	129
9	38.5	20	0.5	78	41	40	4.75	0.325	76.6
10	34.2	20	0.55	69.9	42	57.8	6.3	0.325	97.8
11	28	20	0.6	56.7	43	58.7	12.5	0.325	103
12	23.8	20	0.65	47.2	44	61	20	0.325	142
13	41.3	20	0.5	101	45	55	4.75	0.325	122
14	35.8	20	0.55	88.3	46	63	6.3	0.325	137
15	30.2	20	0.6	75.8	47	75	12.5	0.325	151
16	24.9	20	0.65	59.2	48	74	20	0.325	165
17	38.5	20	0.5	96.2	49	35	2	0.54	49.3
18	53.6	12	0.5	104	50	93	8	0.4	140
19	43.9	12	0.5	104	51	68	12	0.4	126
20	19.8	12	0.5	38.8	52	21	12	0.8	71
21	74	12	0.4	119	53	29	10	0.6	70.5
22	29.8	12	0.7	81.3	54	58.9	10	0.4	80.8
23	55.9	12	0.5	123	55	33.1	10	0.55	76.6
24	54.4	12	0.5	105	56	53.6	12	0.5	104
25	52.7	8	0.5	101	57	43.9	12	0.5	104
26	55.3	16	0.5	111	58	19.8	12	0.5	38.8
27	41.8	9.5	0.5	92.9	59	74	12	0.4	119
28	47.3	12.5	0.5	98	60	29.8	12	0.7	81.3
29	63.49	12.5	0.4	125	61	55.9	12	0.5	123
30	69.73	12.5	0.36	127	62	54.4	12	0.5	105
31	54.73	12.5	0.4	93	63	52.7	8	0.5	101
32	60.17	12.5	0.36	96	64	55.3	16	0.5	111

As previously discussed, considering multiple parameters in the modeling process contributes to more accurate estimations. Based on a review of the relevant literature, the proposed models for predicting the fracture energy of concrete typically include three variables: the 28-day compressive strength of standard concrete specimens ($f_c^{'}$), the maximum size of coarse aggregates (dmax), and the water-to-cement ratio in the concrete mix (W/C). These three variables are considered the key input parameters, while the concrete fracture energy (G_F) is treated as the output variable.

It is worth noting that additional variables were available during data collection. However, due to the importance of the 28-day compressive strength as a representative property of concrete, those variables were excluded from the analysis. Accordingly, the LGP-based model developed in this study can be expressed as a function of the aforementioned variables, as represented in Eq. 1.

$$G_F = f\,(f_c', d_{max}, w/c)) \quad (1)$$

2.3. Statistical analysis of the dataset

Statistical description of the dataset provides insight into the features of the variables incorporated in the modeling process. The analysis includes critical statistical measures such as the number of data points, minimum and maximum values, standard deviation, variance, and mean. The possible evaluation of input and output variable distribution and variability can be understood from these measures. The statistical properties of the dataset used in this study are summarized in Table 2.

Table 2. Statistical Characteristics of the Data.

Parameter	f'$_c$ (MPa)	d_{max} (mm)	W/C	G_F (N/m)
mean	49.35	12.38	0.47	98.38
std	1.99	4.95	0.11	31.08
variance	288.89	24.49	0.01	965.74
minimum	19.8	2	0.29	38.8
maximum	93	20	0.8	180
count	64	64	64	64

To further illustrate the characteristics of the data, histograms displaying the distribution of input and output variables are provided in Fig. 1. These histograms help visualize the spread and concentration of values within the dataset, offering a clearer understanding of the data distribution. In addition, Fig. 2 presents the correlations between different parameters of the test database. This correlation matrix highlights the relationships between variables, revealing patterns that could significantly influence the modeling and prediction processes.

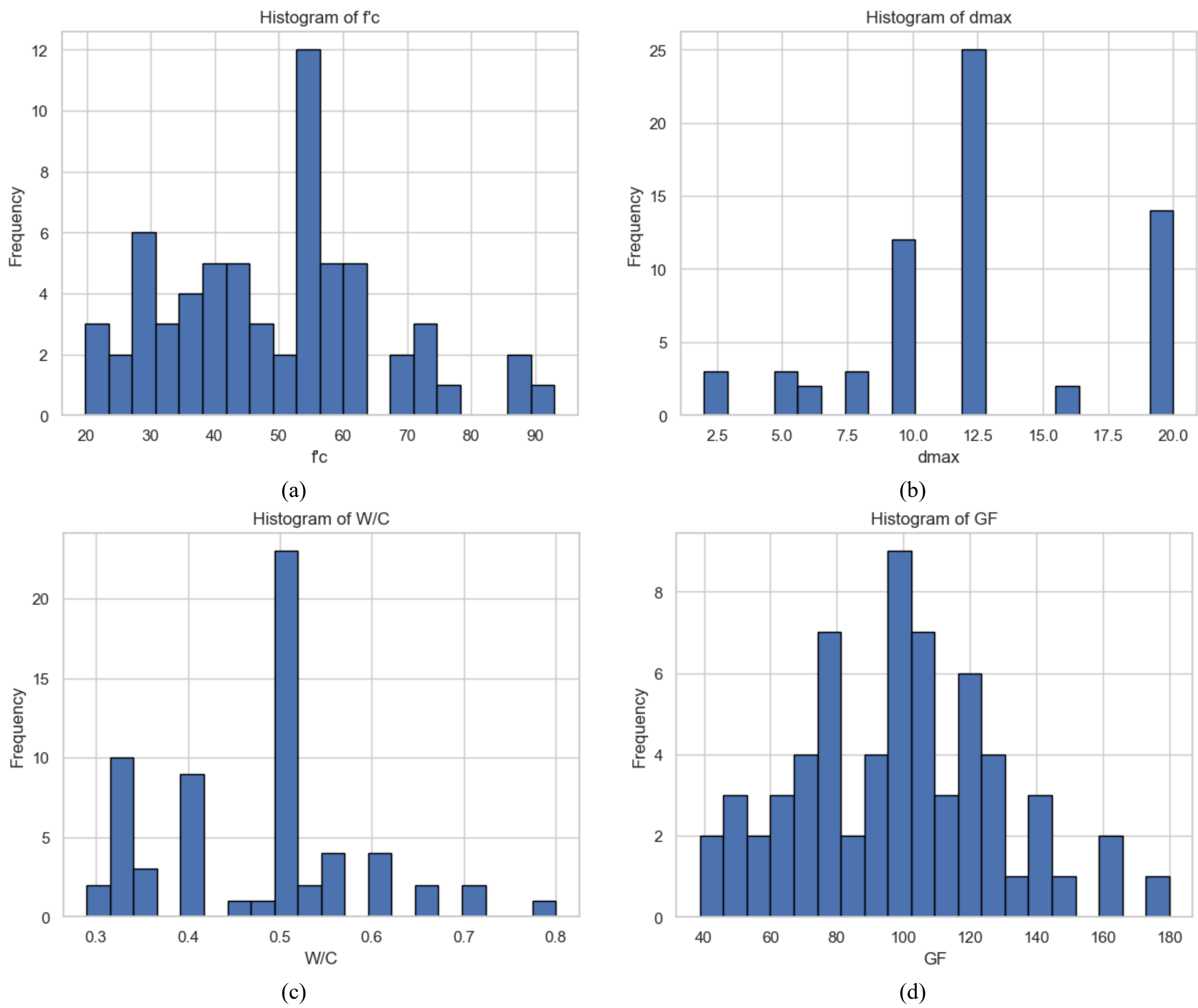


Fig. 1. Histograms displaying the distribution of input and output variables in this study; (a) f'$_c$ (b) $_{dmax}$, (c)W/C, and (d) G_F.

2.4. Evaluation of the developed models' performance

In the evaluation of the performance of the models developed, three very important statistical indicators are utilized: root mean square error (RMSE), mean absolute error (MAE), and correlation coefficient (R). These performance indices give a detailed evaluation of the extent of accuracy and consistency of the model prediction. The parameters can be computed by Eqs. 2, 3, and 4, respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(m_i - t_i)^2}{n}} \tag{2}$$

$$MAE = \frac{\sum_{i=1}^{n}|m_i - t_i|}{n} \tag{3}$$

$$R = \frac{\sum_{i=1}^{n}(m_i - \bar{m})(t_i - \bar{t})}{\sqrt{\sum_{i=1}^{n}(m_i - \bar{m})^2 \sum_{i=1}^{n}(t_i - \bar{t})^2}} \tag{4}$$

In the above equations, m_i and t_i represent the i-th observed (experimental) value and the corresponding predicted value by the final model, respectively. Additionally, $\bar{m}$ and $\bar{t}$ denote the mean values of the observed and predicted datasets, respectively.

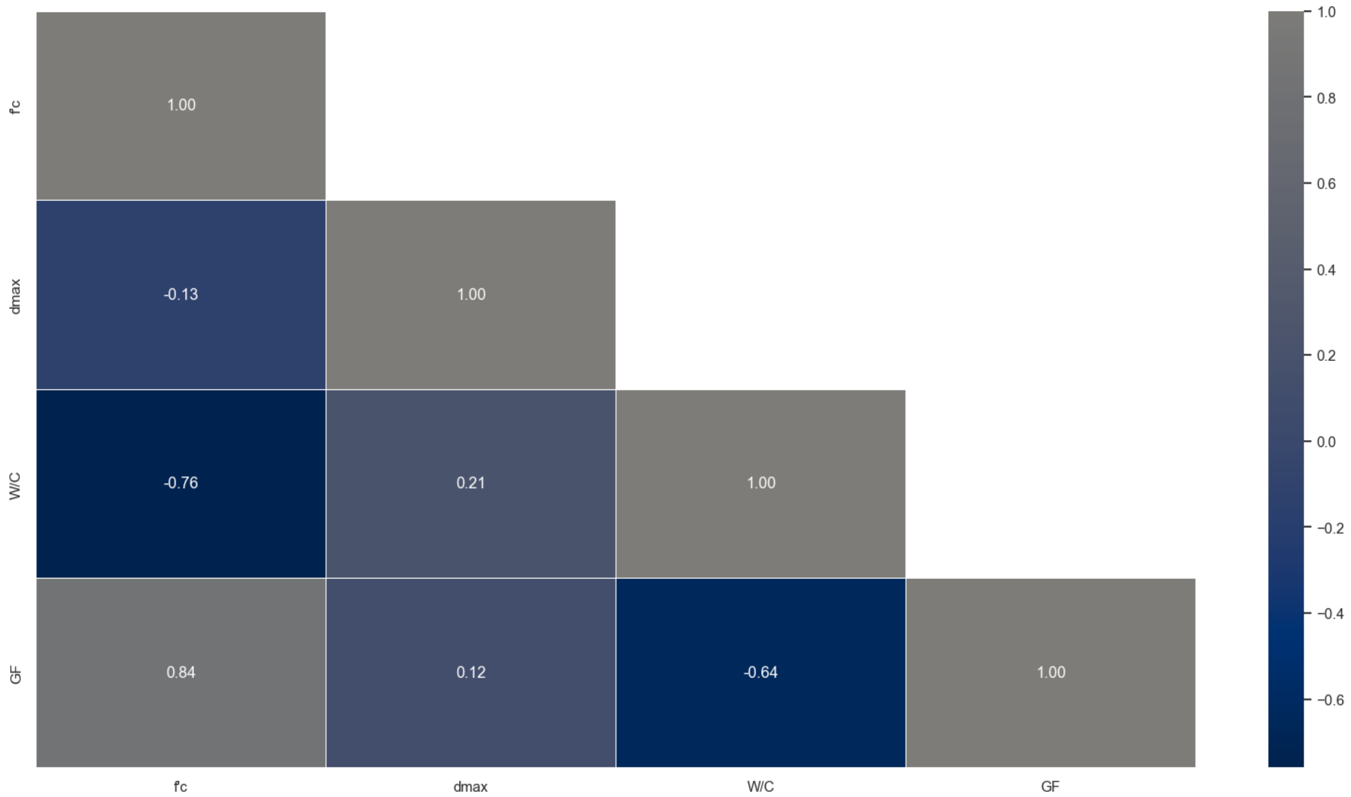


Fig. 2. Correlations between different parameters of the test database.

It is worth mentioning that, based on a reasonable hypothesis, Smith [30] proposed the following criterion for evaluating model performance:

If |R| > 0.8, there is a strong correlation between the predicted and measured values.

If 0.2 < |R| < 0.8, there is a moderate correlation between the predicted and measured values.

If |R| < 0.2, the correlation between the predicted and measured values is weak.

3. Model development using linear genetic programming

To develop the model, a Linear Genetic Programming (LGP) software named Discipulus [31] was utilized. This software employs a genetic programming algorithm to determine an appropriate functional form and to optimize its parameters. LGP in Discipulus is executed through multiple runs, during which the software intelligently adapts its parameters to the specific problem under investigation. One of the key advantages of Discipulus is its use of direct machine-level binary instructions for program execution, which significantly increases its processing speed compared to other automated training methods.

One of the main challenges in soft computing techniques is overfitting—a condition where the developed model performs well on training data but fails to generalize to unseen data [27]. In such cases, while the training error remains low, the model's predictive error on new data becomes high. Overfitting can occur due to complex training responses, extensive training durations, or small training dataset sizes, which reduce the reliability of predictions [28].

To prevent overfitting, once the model is trained on a subset of the data, a separate test set is used to evaluate the model's generalization performance. Therefore, the dataset is typically divided into training and testing portions. The testing data helps assess the final model's robustness and predictive reliability. Based on prior studies in artificial intelligence and expert recommendations, commonly 60–80% of the available data is used for training, while the remaining 20–40% is used for testing [20]. In this study, 75% of the data was used for training and 25% was reserved for testing.

To achieve optimal LGP models for predicting concrete fracture energy, the software was executed approximately 200 times. In each run, the input parameters to the software were varied. After extensive trial and error, the best set of input parameters was identified and is presented in Table 3.

Table 3. Configuration of Parameters in the LGP Algorithm.

Parameter	Settings
Initial Population	500–1000
Maximum Program Length	128–256
Initial Program Length	64
Crossover Rate (%)	50 and 90
Mutation Rate (%)	95
Operators	+,−,×,/,√
Number of Demes (Subpopulations)	10 and 20

As shown in Table 3, to ensure that the final model remains usable for manual computations, only four basic arithmetic operators (+, –, ×, ÷) and the square root function (√) were included in the formulation of the LGP model. To avoid excessive complexity in the evolved programs, maximum program lengths of 128 and 256 were employed. Additionally, demes were used to divide the population into subgroups. In the present model, 10 and 20 demes were tested for population subdivisions. Crossover and migration occurred between adjacent demes, promoting genetic diversity and accelerating the evolutionary process. After the completion of model training and analysis of experimental data, it is now necessary to evaluate the performance of the proposed models. Three statistical measures, namely R, RMSE, and MAE, are thus used for this purpose.

It is worth noting that in genetic programming, input data preprocessing (such as normalization) plays a minimal role. Typically, the input variables are fed into the model directly as observed in the problem domain. This characteristic provides a significant advantage of genetic programming over traditional genetic algorithms, neural networks, and other machine learning algorithms.

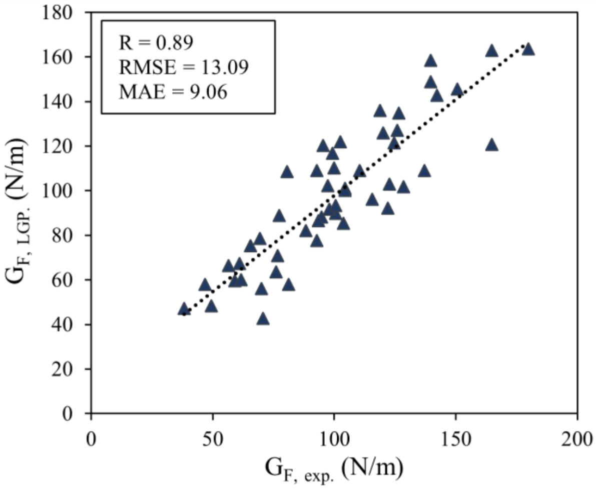
3.1. LGP-based model

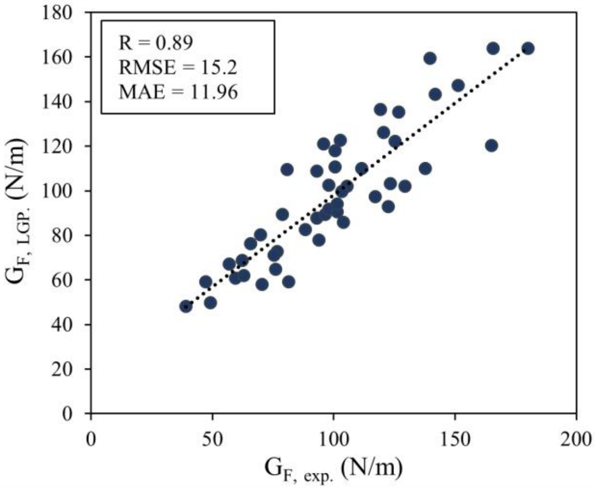
Various LGP models with different parameter settings were developed, and the best-performing LGP model was selected based on the conducted analyses. The outcome of this optimal model for predicting the fracture energy of concrete is presented in Eq. 5.

$$G_F = f_c' + \sqrt{d_{max}\,(f_c' + \frac{\sqrt{0.7\,\frac{{f_c'}^2}{W/C}}}{W/C})} \quad (5)$$

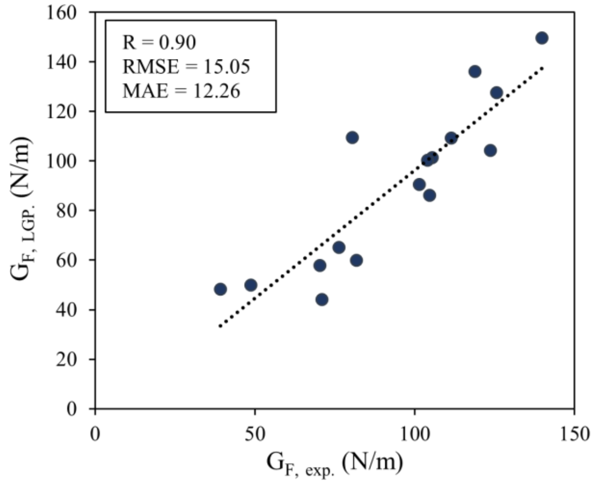
3.2. Evaluation of the accuracy

To assess the model's performance in predicting concrete fracture energy, the predicted results are compared with the experimental data across all data categories—training, testing, and the full dataset—as illustrated in Fig. 3. Additionally, to further evaluate the model's predictive accuracy on various data subsets, the statistical indicators R, RMSE, and MAE are also presented. Fig. 3 clearly shows the correlation coefficients of the whole dataset, training, and test datasets are 0.89, 0.89, and 0.90, respectively. Furthermore, the RMSE and MAE values indicate that Linear Genetic Programming (LGP) and the proposed model are reliable for approximating and predicting the fracture energy of concrete. In addition, the close agreement between the error metrics and correlation coefficients for both training and testing sets confirms the generalization capability of the proposed model and demonstrates that overfitting has not occurred. The model's high accuracy on the test data clearly reflects its strong predictive performance. To benchmark the proposed LGP model, the error analysis parameters for previously developed models used for predicting concrete fracture energy are summarized in Table 4.





(a) (b)



(c)

Fig. 3. Comparison Between LGP Predictions and Experimental Results: (a) All Data, (b) Training Data, (c) Testing Data.

Table 4. Comparison of Error Metrics for Different Models in Predicting Concrete Fracture Energy (for the Entire Dataset).

Parameter	LGP	Bazant & Bec (2002) [7]	CEB (1990) [8]
R	0.89	0.87	0.45
RMSE	13.09	26.78	223.28
MAE	9.06	20.79	154.22

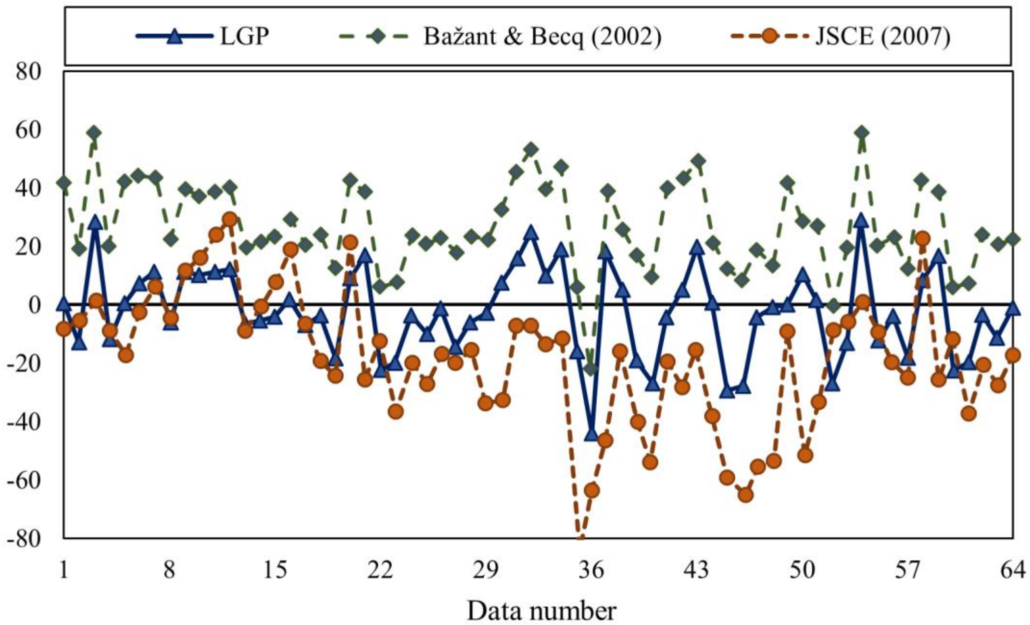


Fig. 4. Comparison of Prediction Errors from Different Methods for the Entire Dataset Used in This Study to Estimate Concrete Fracture Energy

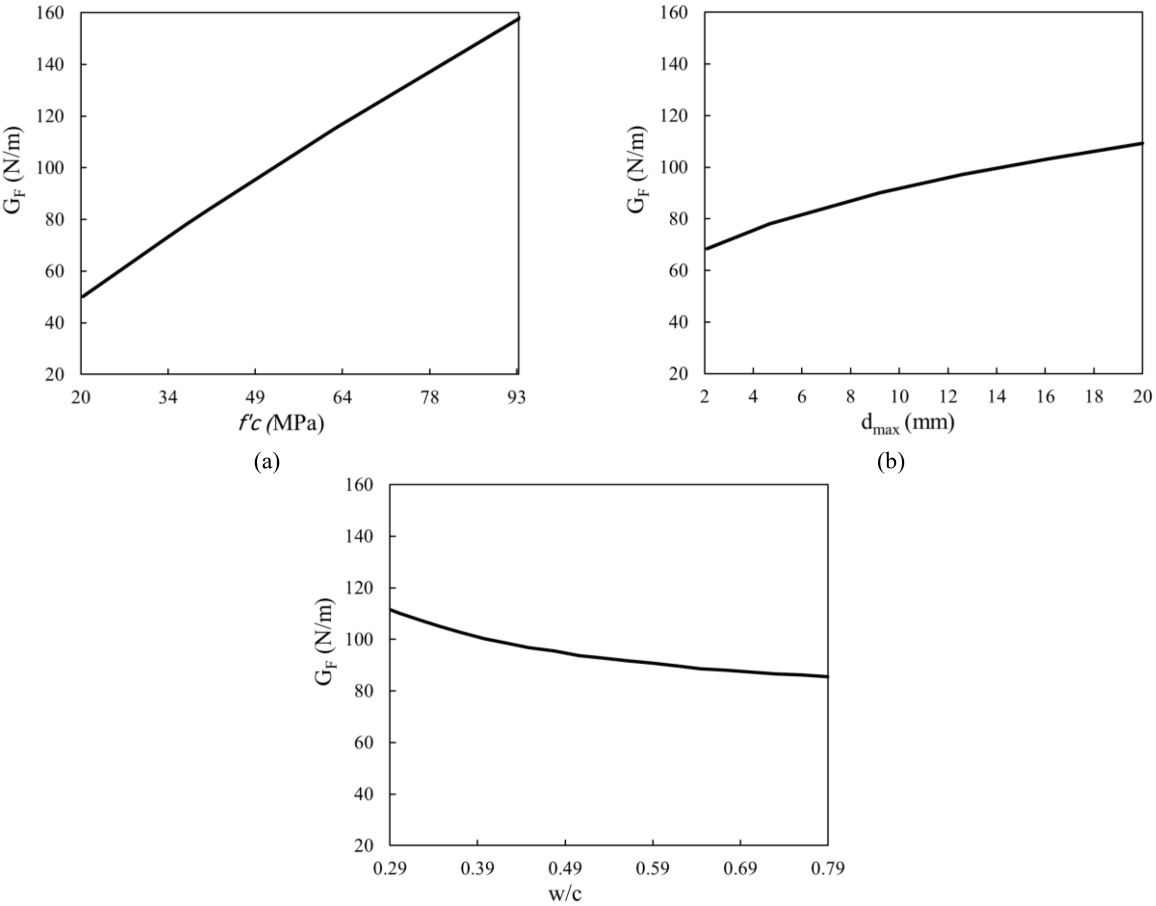
As observed, the prediction error for the compressive strength of self-compacting concrete is significantly lower compared to other models when validated against experimental results. This demonstrates that the proposed approach is capable of accurately predicting the fracture energy of concrete with low error and high precision. It is worth noting that in analytical approaches, such as the limit equilibrium method, the governing equations, and models are derived based on simplifying assumptions. These assumptions are typically introduced to make the problem conceptually manageable. However, such simplifications often result in deviations from real-world conditions and lead to reduced accuracy and increased error.

On the other hand, regression-based and statistical analysis models are usually developed by curve-fitting a few predefined forms to a limited number of experimental data points. These models often fail to generalize well to new data that were not involved in the modeling process. By contrast, one of the strengths of artificial intelligence techniques—particularly the LGP method—is that the resulting model is developed through extensive trial and error [23]. These models are evaluated based on their performance on separate test datasets after training, allowing them to adapt more effectively to real-world complexity. Therefore, the LGP model demonstrates strong predictive capabilities, which can be considered one of the key advantages of this method when modeling complex engineering problems.

3.3. Parametric study of the proposed model

To further investigate the accuracy of the proposed LGP model, a parametric study was conducted using its results. The main objective of this analysis is to determine the influence of each input parameter on the compressive strength and physical behavior of the model. Based on previous studies and the adopted methodology, the parametric analysis was carried out by varying one input parameter at a time while keeping all other variables fixed at their mean values. The results of this parametric study are illustrated in Fig. 5. It is worth noting that, according to the defined relationships and the expected influence of the input parameters on concrete fracture energy, it is anticipated that GF will increase with increasing f'c and dmax, and decrease with an increase in the water-to-cement ratio (w/c).

As shown in Fig. 5, the observed trends for all parameters are consistent with empirical knowledge of physical behavior and the effects of input variables on the model. These findings further confirm the accuracy and reliability of the developed models.



(c)

Fig. 5. Parametric Study of the LGP Model Parameters: (a) f'_c, (b) d_{max}, (c) W/C

4. Conclusion

In this study, a branch of computational intelligence techniques known as Linear Genetic Programming (LGP) was employed to predict the fracture energy of concrete. The proposed LGP model was developed using experimental data extracted from previously published studies, and the best-performing model was identified and formulated. To enhance usability, the resulting model was expressed in a simple and practical formula. The key findings from the developed LGP models are summarized as follows:

- This research, for the first time, investigated and demonstrated the capability of Linear Genetic Programming, an artificial intelligence algorithm, in modeling and predicting a critical structural engineering parameter—concrete fracture energy.
- The prediction results obtained from the LGP models can be effectively utilized for routine design calculations, either manually or through spreadsheet-based programming.
- To simplify the computation process, the most accurate and efficient model was presented in the form of an equation, which delivers acceptable estimates of fracture energy when compared to other existing models.
- Another important feature of the LGP-based models is the high level of interaction between the user and the modeling process. The user's physical understanding of the problem can influence the selection of functional elements and the structure of the resulting models. To illustrate this aspect, a parametric analysis was conducted using the developed LGP model. The results confirmed the physically and mathematically consistent behavior of the model and its strong agreement with empirical results

and experimental observations.

Statements & declarations

Ali Nazari: Conceptualization, Investigation, Methodology, Formal analysis, Resources, Writing - Original Draft, Writing - Review & Editing.

Shahin Lale Arefi: Conceptualization, Methodology, Project administration, Supervision, Writing - Review & Editing.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

Declarations

The authors declare no conflict of interest.

Data availability

The data presented in this study will be available on interested request from the corresponding author.

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