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GMDH-Derived Correlation for Investigating Soil Internal Friction Angle from SPT and Physical Indices

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ABSTRACT

This study develops and evaluates a Group Method of Data Handling (GMDH)–based polynomial model to estimate the internal friction angle (ϕ) of soils from in-situ measurements. The modeling framework uses 318 data sets compiled from multiple geographic regions, thereby capturing broad variability in depositional history and soil fabric. Predictor variables comprise the Standard Penetration Test (SPT) blow count (N) together with seven influential in-situ/geotechnical parameters: effective overburden stress, fines content, Atterberg limits, water content, dry unit weight, and particle-size descriptors, including the percentage passing the No. 4 sieve. The GMDH algorithm automatically selects structure and coefficients, yielding an interpretable polynomial network that balances accuracy and parsimony. Model performance was assessed using standard statistical indicators and external verification procedures, and input significance was examined through sensitivity analysis to quantify the relative contribution of each parameter. Results demonstrate that the proposed model attains high predictive accuracy for ϕ across the heterogeneous, globally sourced database, underscoring its robustness to site-specific biases. Given its reliance on commonly available field and laboratory indices, the approach offers a practical and powerful tool for preliminary design and data-driven characterization of shear strength parameters where direct shear or triaxial testing is limited. Overall, the findings support GMDH as an effective methodology for identifying reliable empirical relationships between ϕ and routinely measured soil properties.

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1. Introduction

Standard Penetration Test (SPT) is extensively employed in geotechnical research and exploration projects to quickly and cost-effectively assess the mechanical characteristics of soil layers. In this test, a 76 mm borehole is drilled, and at specified intervals, a split spoon sampler with a diameter of 50.5 mm is lowered into the borehole and impacted. The number of blows to penetrate 450 mm is registered. The number of blows in the first 150 mm is neglected because of the material disturbance during the drilling process. The SPT N value is therefore the number of blows required to penetrate 300 mm [1]. It should be noted that the results of SPT in granular soils have a limited application. For example, coarse grains of sand and rubble do not enter the SPT sampler, and the obtained blows will not be acceptable. The other limitation of SPT is that it does not provide an index of cohesion due to cementation of granular soils. SPT provides a suitable sample for performing soil-related laboratory tests. SPT N value and soil engineering behavior can be related through local correlations, as well as extensive empirical relationships.

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Identification of soil properties plays an important role in geotechnical engineering, such as soil classification, quantitative and qualitative expression of soil, and estimation of soil behavior during swelling or settlement. The laboratory and field techniques reveal the true value of soil features such as the internal friction angle (ϕ) and cohesion (C). Standard Penetration Test (SPT) and Cone Penetration Test (CPT) are two examples of in-situ tests that are favored for direct determination of the internal friction angle of soil due to the challenges associated with soil collection and the high prices of representative undisturbed samples [2-5].

SPT has three advantages over CPT:

1. SPT supplies a sample of the soil in which the experiment was conducted and, hence, determines the general categorization of the soil, while CPT only has a cone tip and is not capable of doing this. In these situations, soil classification can be determined only based on the correlation between experiment results or by using the findings from adjacent pits and boreholes.

2. In geotechnical investigation projects, SPT is often carried out during drilling boreholes. As a result, it does not add any more costs to the design.

3. The SPT's patented equipment isn't very extensive, and almost all of the test tools are also utilized for drilling. CPT, however, frequently needs expensive proprietary equipment [6-8].

Because determining soil shear strength parameters − particularly in depth − is both costly and time-consuming, it is essential to develop models that can easily forecast these characteristics [9]. Therefore, a reliable correlation based on commonly available SPT and soil index data can provide a practical preliminary estimate of the internal friction angle when direct laboratory shear strength tests are limited. Such estimates may also be useful in seismic geotechnical applications, where preliminary evaluation of bearing capacity and soil-micropile interaction in liquefiable sandy deposits is required. In the literature, various modeling techniques based on multivariate regression and pattern recognition have been reported for system identification. Some of these include genetic algorithms, artificial neural networks, neuro-fuzzy systems, and polynomial classifiers. These techniques employ learning paradigms to assess the system parameters.

When simulating a highly nonlinear system, the application of polynomial classifiers becomes problematic in terms of processing and storage perspectives. Besides, the requirement for high-order polynomials may lead to numerical instability. The Group Method of Data Handling (GMDH) method was then presented. A multilayered second-order polynomial network called GMDH is organized according to a training scenario.

Without prior knowledge of the process, this technique can be applied to model and analyze complex and specific systems where there is an unknown relationship between variables. Recently, GMDH has been successfully used in geotechnical engineering thanks to the utilization of such self-organizing networks [10-16].

GMDH has been used by researchers in geotechnical engineering for different purposes. For example, Naeini et al. [17] predicted the subgrade reaction modulus (K_s) of clayey soil through GMDH. They found that K_s obtained by Plate Load Test (PLT) and GMDH-based equations were in excellent agreement. Moreover, the suggested GMDH model's soil liquid limit (LL) was the strongest factor for forecasting K_s.

Jahed Armaghani et al. [18] used GMDH for the prediction of rock deformation. They concluded that GMDH is a strong and reliable method for predicting rock deformation. Ardakani and Kordnaeij [19] applied GMDH for the prediction of soil compaction parameters. The results demonstrated that GMDH is very good at forecasting soil compaction parameters. Additionally, they discovered that the parameters that have the most influence on soil compaction are LL and PL (soil plastic limit). Kim et al. [5] utilized GMDH for the prediction of undrained shear strength of soil. They found that GMDH is an efficient approach in achieving a new empirical mathematical model to provide an appropriate prediction of the undrained shear strength of soils. MolaAbasi et al. [20] used GMDH to forecast the behavior of cement-treated sands. They strongly recommended GMDH as a potent approach to assess the mechanical properties of soil such as stiffness, failure strain, and brittle index. Related studies on treated/reinforced soils and data-driven prediction approaches in civil engineering have also been reported by Ghadakpour et al. [21], Ghadakpour et al. [22], Kutanaei Saman et al. [23], Nematzadeh et al. [24], and Memarzadeh et al. [25], which provide useful background for the present research.

The purpose for the creation and development of an GMDH artificial neural network in this study is to estimate the internal friction angle, taking into account different soil parameters such as the number of modified blows for the hammer energy ratio of 70% (N'_{70}), effective overburden stress (σ'_v), fine content (FC or F_{200}), LL, PI (plasticity index), water content (w), dry unit weight (γ_d), the soil particle diameter below which 10, 30, 50, and 60% of the particles are smaller than this size (D_{10}, D_{30}, D_{50}, D_{60}) and the percentage passing sieve No. 4 (F_4). The reason for choosing these parameters is based on their effect on the strength and stability of the soil. Some input variables, particularly LL, PI, fines content, and water content, may be interrelated. However, in the GMDH framework, the final polynomial structure is selected based on predictive performance, and the sensitivity analysis provides additional insight into the relative contribution of each parameter. The possible influence of multicollinearity has therefore been acknowledged as a factor that should be considered when interpreting the model. Some of the selected geotechnical input variables may show statistical dependency because soil index properties are often interrelated. For example, fines content, Atterberg limits, water content, and particle-size distribution parameters can be partially correlated depending on soil type and depositional conditions. Therefore, the final GMDH model and the sensitivity analysis results were interpreted with consideration of possible multicollinearity among the input variables. The selected parameters were retained because they are commonly available in geotechnical investigations and represent different aspects of soil behavior, including penetration resistance, stress condition, plasticity, density, water content, and gradation. These parameters were selected because they represent penetration resistance, stress condition, plasticity, density, water content, and particle-size distribution, which are directly or indirectly related to the shear strength behavior of soils. In this regard, at first, the previous studies about the correlation between ϕ and other soil parameters were presented. Then, the modeling method with GMDH was described. Finally, the accuracy of the GMDH model was evaluated using sensitivity analysis. It should be mentioned that several methods have been proposed for the determination of the modified standard penetration number. In this research, the Bowles method was used for this purpose because in the Bowles method, 70% of the energy is transmitted to the sampler considering the advanced equipment of SPT [26].

2. Review of previously proposed correlations and the novelty of this research

A series of studies about the application of the standard penetration blow counts for assessing the internal friction angle of soil are presented in this section. So far, numerous empirical relationships between the internal friction angle of the soil, the results of SPT, and other soil parameters have been presented in various scientific investigations. Table 1 presents a summary of previous investigations and empirical correlations between internal friction angle, standard penetration number, and other parameters.

Table 1. Correlation between SPT number, soil parameters, and internal friction angle.

Row	Researchers	Proposed correlation	Remark
1	Dunham [27]	$\phi_d = (12N)^{0.5} + 25$	Angular and well-graded soil particles
		$\phi_d = (12N)^{0.5} + 15$	Round and uniform-graded soil particles
		$\phi_d = (12N)^{0.5} + 20$	Round and well-graded or angular and uniform-graded soil particles
2	Peck et al. [28]	$\phi_d = 53.881 - 27.6034\exp(0.0147N_{60})$	—
3	Shioi and Fukui [29]	$\phi = \sqrt{18N'_{70} + 15}$	Roads
		$\phi = 0.36N'_{70} + 27$	Bridges
		$\phi = 0.45N'_{70} + 20$	Buildings
4	Wolff [30]	$\phi = 27.1 + 0.3N_{60} - 0.00054{N_{60}}^2$	—
5	Kulhawy and Mayne [31]	$\phi = \tan^{-1}\left(N_{60}\left(12.2 + 20.3(\sigma'/p_a)\right)\right)^{0.34}$	Sandy soils
6	Japan Road Association [32]	$\phi_d = (15N)^{0.5} + 15$	$N > 5$ & $\phi_d < 45°$
7	Hatanaka and Uchida [33]	$\phi_d = (20N_1)^{0.5} + 20$	—
8	Zekkos et al. [34]	$\phi' = 3.5\sqrt{N_{1,60} + 15} + 22.3 \pm \varepsilon$	—
9	Hettiarachchi and Brown [35]	$\phi' = \beta' \tan^{-1}\left(\frac{0.2N_{60}}{K(\sigma'/p_a)} - 0.68B\right)$	Sandy soils
10	Esmaeilzadeh et al. [36]	$\phi = 0.66N'_{70} + 8.52$	Sandy soils
		$\phi = 13.56\ln N'_{70} - 18.20$	
11	Mahmoud [3]	$\phi = 0.209N'' + 19.68$	Silty clay with sand
12	Shooshpasha et al. [37]	$\phi' = -3.574 - 0.183FC + 1.346Y_1 + 0.00111FC^2 - 0.0056{Y_1}^2 + 0.00127Y_1FC$	Granular soils
13	Salari et al. [38]	$\phi = 0.474N + 16.188$	Granular soil (GW)
		$\phi = 0.3556N + 20.703$	Granular soil (GC)
14	Mujtaba et al. [39]	$\phi = 0.7N_{60} + 18$	—
15	Kasim and Raheem [40]	$\phi = 0.8531N + 0.1581$	—
16	Dalai and Patra [41]	$\phi = 8.103N^{0.458}$	Cohesionless soil

Compared with previous studies, the novelties of the current research are its simultaneous treatment of both fine- and coarse-grained soils, ensuring applicability across common engineering materials; the compilation of a globally sourced database (318 data sets) that captures wide variability in depositional and stress histories; the implementation of a two-hidden-layer GMDH architecture that improves representational capacity while retaining interpretability of the polynomial network; and the targeted use of seven routinely measured geotechnical predictors, which collectively elevate prediction accuracy for the internal friction angle. Together, these advances deliver a robust, data-driven correlation that is practical for preliminary design where direct shear or triaxial testing is constrained, yet rigorous enough to generalize across diverse sites and soil types. Although modern machine-learning models such as random forest, XGBoost, support vector regression, and artificial neural networks may also be used for this prediction task, the present study focused on GMDH because it provides an explicit and interpretable polynomial equation that can be readily used in preliminary geotechnical design. A detailed comparison with other machine-learning algorithms is recommended for future studies.

3. The data and case histories

The data of this study were collected from the results of geotechnical tests in different parts of Iran and other countries of the world, including 42 sites and 318 geotechnical data sets. These tests include in-situ and laboratory tests on samples taken from the boreholes. The data collected from the databases are derived from the usual reports of geotechnical studies by prominent and reputable consulting engineers. Fig. 1 shows the geographical range of data on the world map. The database was compiled from different geographical regions and therefore includes variability in depositional history and geological conditions. Nevertheless, regional data imbalance may influence the fitted correlation; therefore, the model should be interpreted as a global preliminary correlation rather than a region-specific design equation. The collected database includes soil records from different geological conditions and covers a broad range of SPT resistance, effective stress levels, soil index properties, and particle-size distributions. These variations help improve the representativeness of the data and support the generalization capability of the proposed correlation within the range of the database.

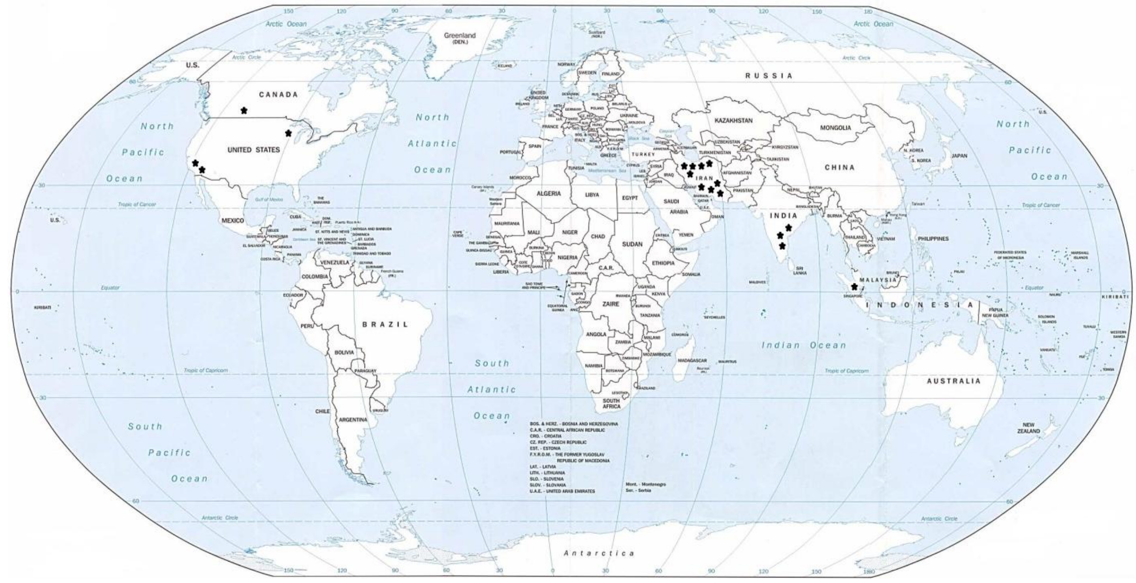

Fig. 1. Geographical distribution of the data on the world map.

It should also be emphasized that many consulting engineers in Iran have used safety hammers in the SPT, and the standard penetration sampler has no inner tube [12]. The data collected from the SPT have relatively similar drilling method and the hammer type. The collected database covers a wide range of SPT values, fines contents, unit weights, water contents, Atterberg limits, particle-size indices, and measured internal friction angles, which supports the development of a preliminary predictive correlation. Furthermore, direct shear test was carried out in the laboratory for determination of the shear strength parameters. Table 2 presents some of the data collected in this research, which are the results of reports by trusted geotechnical consultants.

To provide a clearer overview of the database, a statistical summary of the sample data presented in Table 2 is provided in Table 3. As shown in Table 3, the sample data cover a relatively wide range of corrected SPT values, fines contents, unit weights, water contents, Atterberg limits, particle-size indices, and measured internal friction angles. Therefore, the proposed correlation should be applied with consideration of the range of variables represented in the database. The proposed GMDH correlation should be applied with consideration of the range of input variables represented in the database summarized in Table 3. Predictions made outside these ranges should be considered extrapolations and may not provide reliable estimates for practical geotechnical design. The proposed model was developed using a database containing different soil categories; however, its application should remain within the range of soil types represented in the database. For practical use, the model should be applied with caution to soil categories or geological conditions that are not well represented in the collected data.

4. Description of modeling using GMDH

GMDH is a supervised machine learning tool that was suggested by [42, 43]. This tool is able to automatically synthesize the models. Moreover, it includes several processes such as planting, breeding, duplicating, and selecting or rejecting input variables, setting the model structure and its parameters, and the selection of the model based on the termination principle [42, 44].

With the help of the self-organizing GMDH algorithm, sophisticated models can be created by evaluating how well they perform on a set of multi-input, single-output data pairs (x_i, y_i) $(i = 1, 2, \ldots, M)$. The fundamental concept of GMDH is the creation of an analytical function in a progressive network according to a second-order transmission function, the coefficients of which are acquired by regression analysis [45].

Flexible structure is one of the characteristics of the GMDH network, which can be combined with repetitive and evolutionary algorithms such as back-propagation, genetic programming, particle group optimization, and genetic algorithm [13, 46-52].

Using GMDH, a model can be described as a group of neurons connected in different pairs in each layer by a second-order polynomial. As a result, in the following layer, new neurons are produced. For plotting inputs to outputs, this representation can be utilized. The important point is the challenge of finding a function (f), which can be used roughly instead of observations of the f.

A successful prediction should have an output (y) that is as close as possible to the observed output (y') for a given input vector $X = (x_1, x_2, x_3, \ldots, x_n)$.

Hence, according to M observations, a multi-input and single-output data set is obtained as Eq. 1.

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in})\ (i = 1, 2, 3, \ldots, M) \quad (1)$$

Table 2. A sample of the database extracted from 318 data sets.

N'_{70}	σ'_v (kg/m²)	F_{200} (%)	F_4 (%)	γ_d (kg/m³)	w (%)	PI (%)	LL (%)	D_{10} (mm)	D_{30} (mm)	D_{50} (mm)	D_{60} (mm)	ϕ (°)
5	3381	17.4	99.8	1550	26.30	0	0	0	0.18	0.26	0.35	32.0
3	4415	55	0	1440	32.00	10.00	29.00	0	0.01	0.052	0.08	31.0
12	5925	1	100	1490	23.31	0	0	0.111	0.172	0.212	0.235	27.8
49	5730	95	100	1620	18.00	30.00	31.50	0.003	0.02	0.05	0.12	24.0
58	6300	89	100	1890	16.90	15.00	35.00	0	0.006	0.018	0.025	20.1
11	2250	38.1	97	1370	24.50	0	0	0	0	0.14	0.18	33.0
18	14000	91	96	1700	14.00	15.00	32.00	0	0.005	0.018	0.033	5.0
19	7350	5.8	87	1700	2.00	0	0	0.12	0.6	1.6	2.1	35.0
14	1920	95	0	1510	25.70	26.00	51.00	0	0.0063	0.015	0.026	11.0
7	4936	29	97	1670	19.00	23.00	49.00	0	0.14	0.25	0.8	0.0
40	15600	92	100	1730	14.00	21.00	42.70	0	0.06	0.08	0.09	6.9
6	5730	95	0	1470	30.40	18.00	39.00	0	0.0015	0.005	0.008	3.0
6	30240	85	100	1830	23.91	33.00	57.00	0	0	0.002	0.004	13.0
7	7920	35	64	1600	14.00	9.00	28.00	0.008	0.06	0.9	2.833	21.0
6	19700	100	100	1630	26.00	11.00	34.00	0	0.005	0.0095	0.013	4.0
21	10192	89	99.5	1710	11.00	14.00	23.50	0	0.003	0.005	0.02	28.0
4	3800	95.5	100	1110	49.34	20.00	56.00	0	0	0.00	0.00	22.9
3	3534	4.6	99.9	1000	51.06	0	0	0.09	0.16	0.22	0.25	36.5
22	6152	7	87.5	1800	3.50	0	0	0.13	0.7	1.5	1.8	33.0
21	17820	24	49	1720	10.50	12.00	29.00	0.05	0.271	5	8.158	25.0
9	6000	95	98	1660	25.00	15.00	33.00	0	0.004	0.009	0.014	5.0
4	8700	63	88	1490	27.00	10.00	23.00	0.006	0.046	0.06	0.072	16.0
13	3652	0.3	100	1600	19.85	0	0	0.12	0.177	0.217	0.237	30.6
12	13790	95	100	1660	20.00	15.50	35.00	0	0.005	0.015	0.022	19.0
7	2995	92	100	1250	39.25	0	0	0	0	0.00	0.00	15.8
7	3900	86	0	1580	23.00	11.00	33.00	0	0.003	0.0085	0.015	7.0
16	5610	62	86	1590	17.00	6.00	26.00	0.006	0.037	0.07	1.496	7.0
13	10781	50	100	1460	30.00	29.00	59.00	0	0	0.16	0.3	0.0
12	10440	99	100	1660	25.00	13.00	32.00	0	0.004	0.0085	0.014	14.0
20	3748	0.1	100	1630	20.89	0	0	0.126	0.18	0.219	0.238	29.8
5	13890	58	79	1590	26.00	5.00	23.00	0.005	0.047	0.065	0.14	15.0
7	8510	86	0	1560	19.00	30.00	50.00	0	0	0.0013	0.0023	29.0

Table 3. Statistical summary of the sample database values presented in Table 2.

Variable	Symbol	Unit	Minimum	Maximum	Mean	Standard deviation
Corrected SPT blow count	N'_{70}	-	3.00	58.00	14.28	12.91
Effective overburden stress	σ'_v	kg/m²	1920.00	30240.00	8403.47	6130.41
Fines content	F_{200}/FC	%	0.10	100.00	60.31	36.78
Passing sieve No. 4	F_4	%	0.00	100.00	78.99	36.33
Dry unit weight	γ_d	kg/m³	1000.00	1890.00	1570.94	188.87
Water content	w	%	2.00	51.06	22.73	10.64
Plasticity index	PI	%	0.00	33.00	12.23	10.35
Liquid limit	LL	%	0.00	59.00	26.58	19.41
Particle size at 10% finer	D_{10}	mm	0.00	0.13	0.02	0.05
Particle size at 30% finer	D_{30}	mm	0.00	0.70	0.09	0.17
Particle size at 50% finer	D_{50}	mm	0.00	5.00	0.35	0.94
Particle size at 60% finer	D_{60}	mm	0.00	8.16	0.62	1.54
Internal friction angle	ϕ	Degree	0.00	36.50	18.76	11.43

Then, GMDH should be trained to forecast output values ($\hat{y}_i$) for each given input vector $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ as shown in Eq. 2.

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})\ (i = 1, 2, 3, \dots, M) \tag{2}$$

A reference, a general relationship between the input and output variables, is created by GMDH in the form of a mathematical description. Finding the GMDH network that minimizes the square of the differences between the actual and anticipated outputs is now the key challenge, that is shown in Eq. 3.

$$\sum_{i=1}^{M}\left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) - y_i\right]^2 \to min \tag{3}$$

A sophisticated discrete form of the Volterra function can be used to express the general relationship between input and output variables. Eq. 4 represents a series of this function, which is referred to as Kolmogorov-Gabor polynomials [35, 36, 42, 47].

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \cdots \tag{4}$$

In this investigation, the second-order polynomial in the GMDH network is applied as shown in Eq. 5.

$$\text{Quadratic: } \hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \tag{5}$$

Therefore, to establish the general mathematical relationship between outputs and inputs in Eq. 4, a partial second-order polynomial is applied in a network of connected neurons during a back calculation method. Regression methods are used to derive the coefficients ai in Eq. 5 so that for each pair of x_i, y_i as input variables, the difference between the observed (y) and the calculated ($\hat{y}$) output is minimized.

The second-order form provided in Eq. 5 is typically used to create polynomial trees, and the least squares approach is applied to achieve the coefficients. In order to fit the output in the entire set of input-output data pairs, the coefficients of each G_i quadratic function are generated as shown in Eq. 6.

$$E = \frac{\sum_{i=1}^{M}(y_i - G_i(\ ))^2}{M} \to min \tag{6}$$

The regression polynomial in the form of Eq. 5 that best fits the dependent observations ($y_i = 1, 2, \dots, M$) in least squares is established using the GMDH by taking into account all potential combinations of two independent variables out of the total input variables (n).

In the first hidden layer of the predictor network of observations $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \dots, M)\}$ for various $p, q \in \{1, 2, \dots, n\}$ will therefore $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons be formed. As a result, it is currently possible to establish M data triples $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \dots M)\}$ from observations through $p, q \in \{1, 2, \dots, n\}$, as shown in Eq. 7.

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ x_{Mp} & x_{Mq} & y_M \end{bmatrix} \tag{7}$$

The following matrix equation can be utilized with the employment of the quadratic equation (Eq. 5) for each row of M (Eq. 8).

$$Aa = Y \tag{8a}$$

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5\} \tag{8b}$$

$$Y = \{y_1, y_2, y_3, \dots, y_M\}^T \tag{8c}$$

where Y denotes a vector of the output values of the observations and a is a vector of unidentified coefficients for a quadratic polynomial in Eq. 5. Eq. 9 can be discovered as follows:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \tag{9}$$

When performing a multiple regression analysis, the normal equations are solved using the least squares method (Eq. 10).

$$a = (A^T A)^{-1} A^T Y \tag{11}$$

The vector of the best coefficients from Eq. 5 is determined for the entire set of M, by Eq. 11. It must be clarified that, in accordance with the connection topology of the network, this process is repeated for each neuron in the subsequent hidden layer.

However, a solution derived directly from normal equations is more prone to round-off errors and, more critically, to the singularity of these equations [53]. In a GMDH algorithm, there are six coefficients per estimated model. The coefficients are determined by regular least squares estimation [54]. The flowchart of the GMDH algorithm has been depicted in Fig. 2.

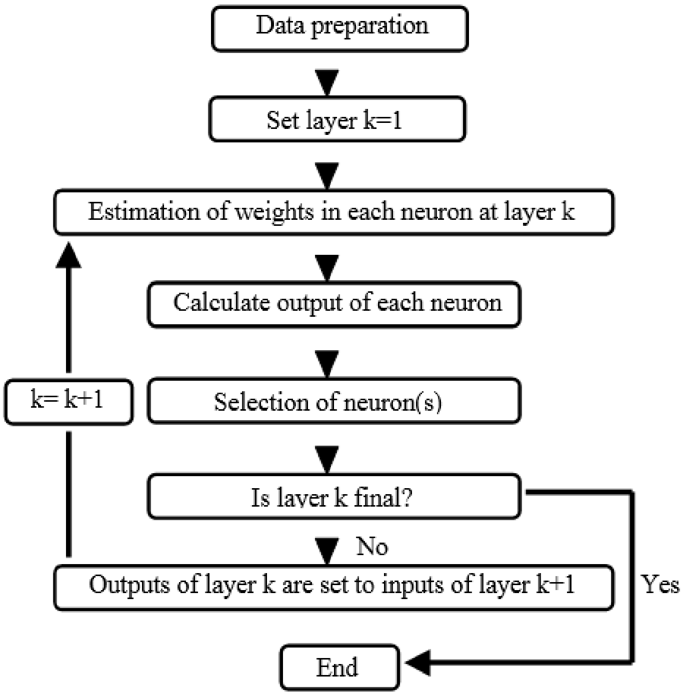

Fig. 2. Flowchart of GMDH algorithm.

5. New proposed model

Training and testing data are separated into two groups in the database. The data trained in the mentioned neural network include 200 input-output data pairs. The complete database was divided into training, testing, and validation subsets, and the validation subset was used only for the external evaluation of the final proposed model. Testing data include 86 input-output data which have not been applied in the prediction training cycle and are only trained to test GMDH models. It is necessary to mention that the selection of training and testing data sets is random, and these data sets have almost identical statistical characteristics. The training and testing subsets were selected randomly with comparable statistical characteristics. To further improve robustness assessment, future work may apply k-fold cross-validation or repeated random subsampling using larger extended databases.

Fig. 3 depicts the performance of model prediction versus observed data for the training and testing data sets. As observed, the predicted and measured values from the training and testing sets are not significantly different. The main trends obtained from the model are generally consistent with established geotechnical understanding. An increase in corrected SPT resistance usually reflects denser or stronger soil conditions and is therefore expected to be associated with a higher internal friction angle. In contrast, higher plasticity and fines-related characteristics may reduce the frictional component of shear strength. The influence of effective overburden stress should be interpreted carefully, as it may also depend on soil fabric, depositional history, and stress history.

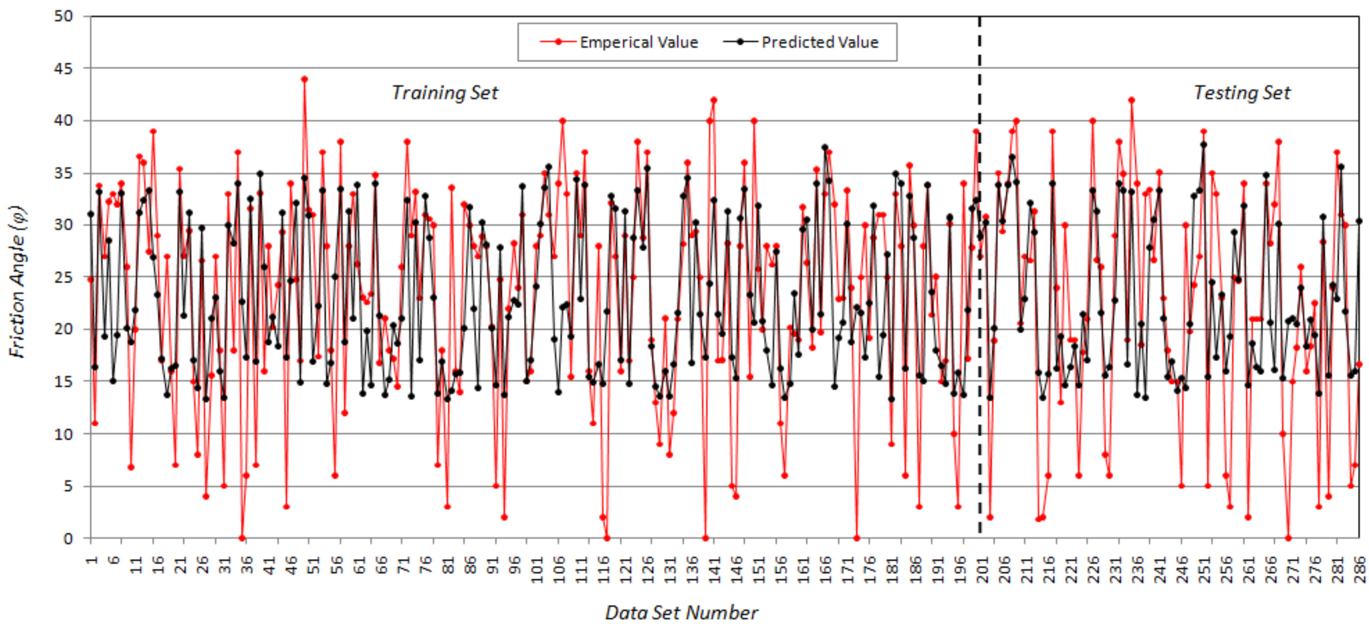

Fig. 3. Comparison between empirical and predicted values of ϕ for training and testing sets.

The developed architecture of GMDH for this research is shown in Fig. 4.

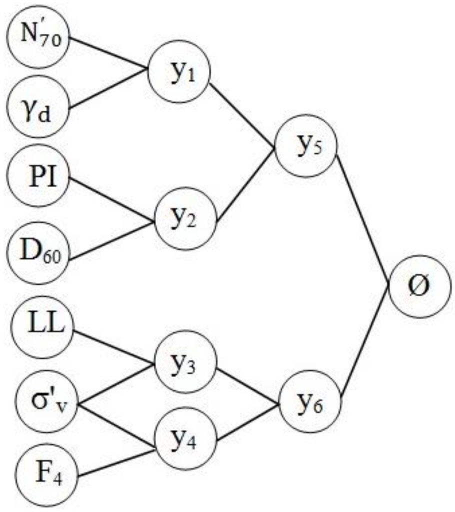

Fig. 4. The evolved architecture of a generalized GMDH for internal friction angle.

Such a model for the internal friction angle has a polynomial representation that is shown in Eq. 12.

$$\phi = -2.350050 + 0.005795y_5 + 0.957297\, y_6 - 0.005976\, {y_5}^2 - 0.024938{y_6}^2 + 0.037769y_5y_6 \quad (12a)$$

$$y_1 = 0.0828 + 0.018199N'_{70} + 1.303645\gamma_d - 0.000001{N'_{70}}^2 - 0.019810{\gamma_d}^2 - 0.000578N'_{70}\gamma_d \quad (12b)$$

$$y_2 = 29.565296 - 0.571619PI + 0.018102D_{60} + 0.006224PI^2 - 0.000011{D_{60}}^2 + 0.000480PID_{60} \quad (12c)$$

$$y_3 = 62.153586 - 62.908345LL + 58.309302\sigma'_v + 22.280136LL^2 - 23.030937{\sigma'_v}^2 + 13.101200LL\sigma'_v \quad (12d)$$

$$y_4 = 73.990484 - 73.453004\sigma'_v + 26.393534F_4 + 29.256098{\sigma'_v}^2 + 24.005152{F_4}^2 - 40.105051\sigma'_vF_4 \quad (12e)$$

$$y_5 = 4.011870 - 0.578298y_1 + 1.302297y_2 + 0.008694{y_1}^2 - 0.007880{y_2}^2 + 0.003496y_1y_2 \quad (12f)$$

$$y_6 = -39.569747 + 4.094071y_3 - 0.178640y_4 - 0.028284{y_3}^2 + 0.048508{y_4}^2 - 0.070782y_3y_4 \quad (12g)$$

The accuracy of the statistical evaluation of this model is determined through MSE (mean squared error), RMSE (root mean square error), R^2 (coefficient of determination) and MAD (mean absolute deviation) according to the following relationships:

$$R^2 = 1 - \left[\frac{\sum_{i=0}^{M}\left(Y_{i(Model)} - Y_{i(Actual)}\right)^2}{\sum_{i=1}^{M}\left(Y_{i(Actual)}\right)^2}\right] \quad (13)$$

$$MSE = \left[\frac{\sum_{i=0}^{M}\left(Y_{i(Model)} - Y_{i(Actual)}\right)^2}{M}\right] \quad (14)$$

$$MAD = \left[\frac{\left|\sum_{i=0}^{M}\left(Y_{i(Model)} - Y_{i(Actual)}\right)\right|}{M}\right] \quad (15)$$

$$RMSE = \left[\frac{\sum_{i=0}^{M}\left(Y_{i(Model)} - Y_{i(Actual)}\right)^2}{M}\right]^{\frac{1}{2}} \quad (16)$$

In Eq. 12, N'_{70} is the corrected SPT blow count, σ'_v is the effective overburden stress, γ_d is the dry unit weight, LL is the liquid limit, PI is the plasticity index, D_{60} is the particle size corresponding to 60% finer, F_4 is the percentage passing sieve No. 4, and y_1 to y_6 are intermediate variables generated by the GMDH network. Table 4 presents the values of these parameters in the current study:

Table 4. Statistical information of the model.

Set	R^2	RMSE	MAD	MSE
Training	0.99	6.52	3.95	42.55
Testing	0.98	6.43	3.74	41.32

RMSE and MAD are reported in degrees, while MSE is reported in squared degrees. All error indices were calculated from the difference between the measured and predicted internal friction angles. The reported RMSE values were checked to ensure consistency with the corresponding MSE values. When GMDH is being designed, it should be remembered that in each layer of the network, the neurons' polynomials are all similar to each other and the network is designed based on similar processes [55]. The optimal performance of the polynomial model for the prediction of a set of data is carried out based on the statistical evaluation of the training and testing data sets (according to Table 4).

6. Validation and sensitivity analysis

The suggested model's validity and accuracy in predicting ϕ were compared with the correlations previously proposed by some researchers [3, 28-30, 33, 36, 38]. This comparison was performed for all data items considered for validation purposes in this section. This set of data is separate from the testing and training data set. Fig. 5 demonstrates the dispersion chart for the measured and predicted values of ϕ. It should be noted that the sensitivity analysis represents the relative influence of the input variables within the developed GMDH model and does not necessarily indicate direct causal relationships. Because some geotechnical variables may be correlated, the apparent importance of one parameter may partly reflect its interaction or dependency with other input variables.

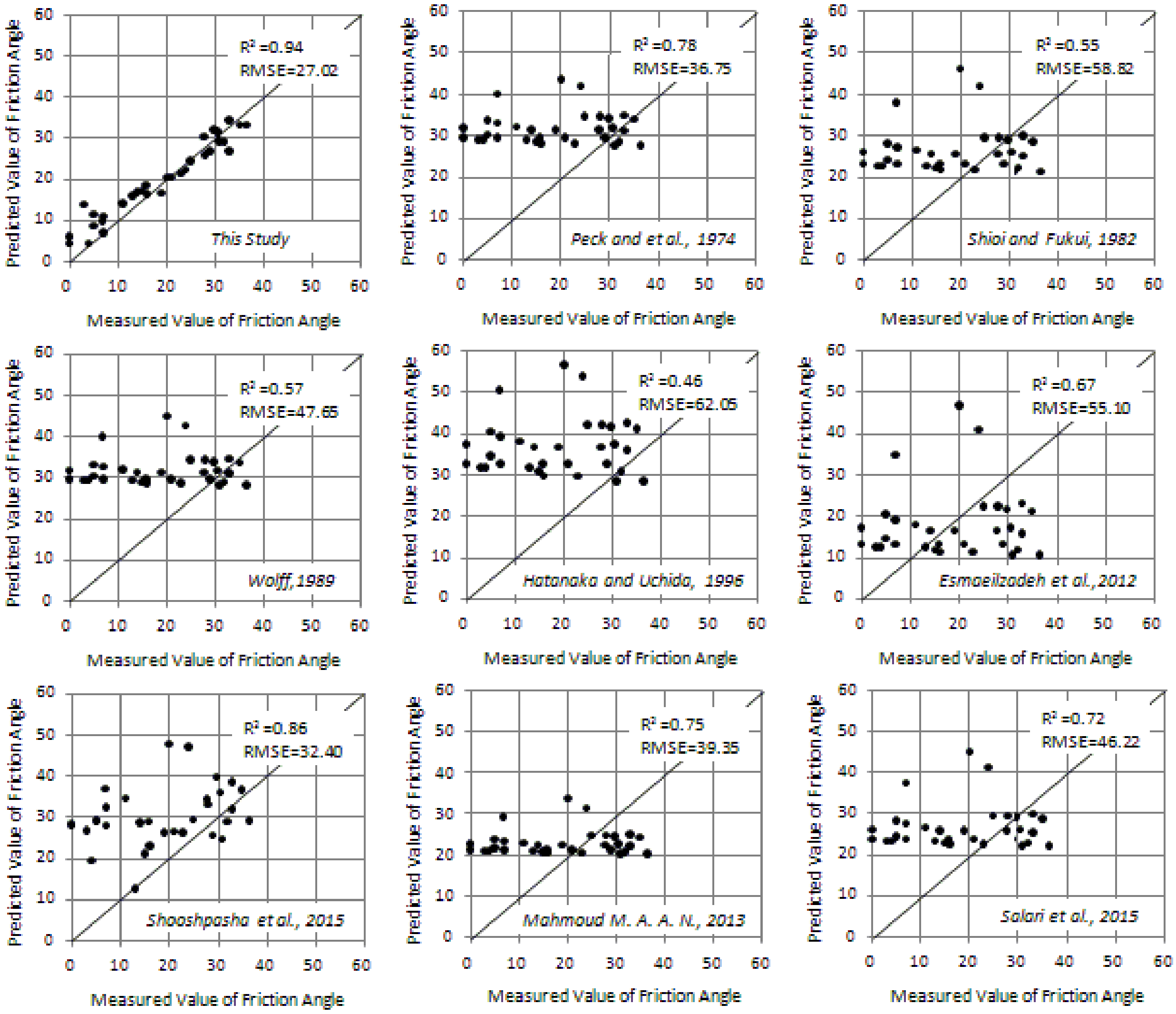

Fig. 5. Comparison of predicted and measured values of internal friction angle presented by different researchers.

The dispersion chart is a useful way to show the relationship between the data. This chart shows cases that have a positive and strong impact on the regression line. In other words, it indicates the cases in which the data are far from the range or focused. Also, a curve or any non-linear relationship can be deduced from this chart. As seen, some researchers have predicted the values of soil internal friction angle higher or lower than the measured values. The GMDH model in the present study predicts the internal friction angle with high precision. The analysis of the overall trend of points shows that there is a linear and positive relationship between variables. In addition, the focused points around the regression line are relatively good, and there is no deviation between the points. Although the present study reports deterministic prediction indices, the scatter of measured and predicted values provides a preliminary indication of prediction uncertainty. Future studies may further enhance the reliability of the proposed model by developing prediction intervals or confidence bounds for practical design applications. Sensitivity analysis in this study was performed in a similar method with partial derivatives [54]. The basis of this analysis is the partial derivative of the output of Y_j for the input x_i which represents a mathematical definition that reflects the sensitivity of Y_j to x_i [56].As a result, a function has a stronger sensitivity to a specific input variable, which causes a greater proportion of its partial derivative to be related to that variable. Also, the impact of input changes on the final output values of a network can be expressed using this method. For example, if the partial derivative value of a variable is positive at a particular point, consequently, a small increase in the input variable will result in a rise in the network's output. Additionally, the model's sensitivity analysis can be utilized to determine how input parameters affect the model's output. This mechanism involves changing each input neuron individually at a steady rate while maintaining other factors constant. Constant variables with the rates of -0.1, 0.05, 0, $+0.05$ and $+0.1$ were selected for the present research. The percentage change in the output for each input neuron is seen as a result of the input neuron's change. The sensitivity of each input neuron is found using Eq. 17 [57]:

$$\text{Sensitivity level of } X\ (\%) = \frac{1}{M}\sum_{j=1}^{M}\left(\frac{\%\ Change\ in\ Output}{\%\ Change\ in\ Input}\right)_i \times 100 \tag{17}$$

Fig. 6 depicts the results of the model's sensitivity analysis. It is clear that ϕ is significantly affected by N'_{70}. Also, LL has the least effect on the value of internal friction angle. The dominant influence of N'_{70} is reasonable from a geotechnical perspective because it directly reflects penetration resistance, relative density/consistency, and the in-situ strength condition of soil. In contrast, parameters such as LL and PI describe plasticity characteristics and may influence the friction angle more indirectly through soil fabric, fines content, and water-soil interaction.

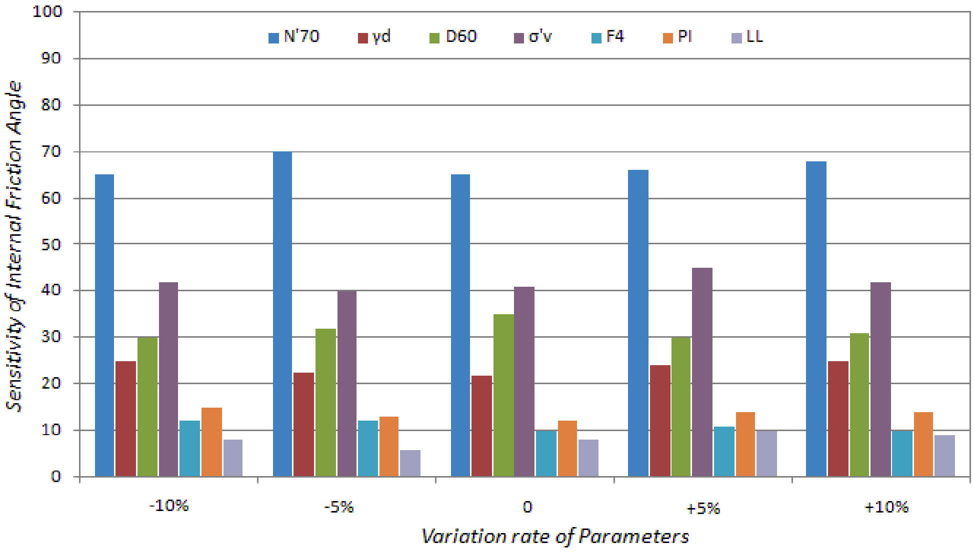

Fig. 6. Comparison of predicted and measured values of internal friction angle presented by different researchers.

7. Summary and conclusions

This research presents a new approach for predicting internal friction angle according to SPT results and geotechnical properties of soil. In order to achieve the proposed model for the internal friction angle in the current research, the developed GMDH neural network has been employed. Moreover, a polynomial mathematical model was proposed for the internal friction angle of the soil based on seven parameters including N'_{70}, LL, PI, γ_d, σ'_v, D_{60} and F_4. The new proposed model was compared with the results of other researchers, and its validation was evaluated. The main trends obtained from the model are generally consistent with established geotechnical understanding. An increase in corrected SPT resistance usually reflects denser or stronger soil conditions and is therefore expected to be associated with a higher internal friction angle. In contrast, higher plasticity and fines-related characteristics may reduce the frictional component of shear strength. The influence of effective overburden stress should be interpreted carefully, as it may also depend on soil fabric, depositional history, and stress history. The results show that the prediction of ϕ has low variance and high accuracy. The model's sensitivity analysis was also performed to examine the impact of the input parameters on the model's output. The results of the sensitivity analysis showed the effectiveness of input parameters for the initial prediction of internal friction angle and also reflected the fact that the geotechnical properties of the soil play an important role in the prediction of ϕ. It was found that ϕ was significantly influenced by N'_{70} and also it decreases with increasing PI, F_4 and LL. In other words, the internal friction angle has the highest and lowest dependency on N'_{70} and LL, respectively. Therefore, the proposed GMDH-based correlation can be considered a practical preliminary estimation tool for geotechnical applications, provided that it is used within the range of soil conditions represented in the database. Future research may extend the proposed GMDH framework to post-liquefaction soil-property estimation and to the seismic performance assessment of micropile foundations in liquefiable sandy deposits.

Statements & declarations

Author contributions

Hassan Esmaeilzadeh: Data curation, Investigation, Formal analysis, Writing - Original Draft, Writing- Reviewing & Editing.

Issa Shooshpasha: Project administration, Supervision.

Hossein MolaAbasi: Data Curation, Investigation, Formal analysis, Writing - Original Draft, Writing - Reviewing & Editing.

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Data availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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