

Numerical Investigation of Reinforced Concrete Deep Beams with Square Web Openings: Influence of Opening Configuration and Reinforcement Detailing

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ABSTRACT

Web openings are frequently introduced into reinforced concrete deep beams to accommodate utility services; however, these discontinuities may significantly disrupt the internal load-transfer mechanism and adversely affect structural performance. This study investigates the influence of square opening size and location on the behavior of reinforced concrete deep beams using nonlinear finite element analysis in ABAQUS. Unlike previous studies that primarily focused on individual parameters such as opening geometry, opening location, or strengthening techniques, the present study systematically investigates the combined effects of square opening size, vertical opening location, and reinforcement detailing on the structural behavior of reinforced concrete deep beams. The numerical model was developed based on the concrete damaged plasticity (CDP) approach and validated against previously published experimental results, showing good agreement in terms of load-displacement response, crack patterns, and ultimate load capacity. Six deep beam specimens containing square web openings with side lengths of 200 and 250 mm positioned at various levels within the shear span were analyzed. The effects of opening configuration on crack propagation, principal stress distribution, and load-carrying capacity were evaluated. The results showed that openings located at the mid-depth of the web produced the greatest disturbance to the main compression strut, leading to increased stress concentration, extensive diagonal cracking, and reduced structural capacity. Increasing the opening size further intensified these effects by disrupting a larger portion of the principal compression strut. To mitigate these adverse effects, a modified reinforcement arrangement was proposed by redistributing web reinforcement around the opening and adding confinement reinforcement in critical regions. This detailing increased the ultimate load capacity by approximately 3-10%, delayed crack initiation by up to 44%, and improved principal stress redistribution. Overall, the proposed reinforcement arrangement effectively enhanced the structural behavior of deep beams with square web openings and provides practical guidance for reinforcement design in disturbed regions.

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1. Introduction

Reinforced concrete deep beams constitute a distinct class of structural members that are widely used in transfer girders, pile caps, foundation components, and coupled wall systems. Unlike slender beams, deep beams exhibit a highly nonlinear strain

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distribution over the depth of the section. Consequently, the Bernoulli hypothesis is no longer applicable, and their behavior cannot be adequately described using classical flexural theory. The theoretical basis of this behavior originates from Saint-Venant's principle, which explains the formation of disturbed regions (D-regions) and the disruption of stress trajectories in the vicinity of load application points and supports. Based on this concept, the strut-and-tie model (STM) was developed as an efficient approach for the analysis and design of deep beams [1] and was subsequently incorporated into major international design codes, including ACI 318 [2]. Within this framework, concrete compressive fields (struts) and steel tensile members (ties) interact through nodal zones to establish the internal load-transfer mechanism of the member.

Nevertheless, in practical applications, the provision of web openings is often unavoidable due to architectural requirements and the need to accommodate mechanical and electrical services. Among the various opening configurations, rectangular (including square) openings are the most commonly adopted because of their ease of construction and compatibility with utility ducts [3]. The introduction of such openings significantly disrupts the ideal load-transfer mechanism represented by the strut-and-tie model [4-9]. When a rectangular opening intersects the natural compressive struts within the shear region, severe stress concentrations develop at its sharp corners, resulting in a substantial reduction in stiffness, shear capacity, and ultimate strength of the member. Furthermore, the initiation and propagation of cracks are markedly altered, leading to a more pronounced and aggressive deterioration of the structural response.

Early investigations by Ray [10] demonstrated that large web openings located within the load-transfer zone between the applied load and the support can significantly reduce the ultimate load-carrying capacity of deep beams by disrupting the internal force-transfer mechanism. Similarly, El Maaddawy and Sherif [4] conducted experimental studies on reinforced concrete deep beams with openings and proposed empirical relationships for predicting shear strength. Their findings highlighted that the structural response is primarily governed by the extent of disturbance imposed on the natural stress flow paths. Regarding the influence of opening geometry, Kumari et al. [11] reported that rectangular and square openings produce substantially greater reductions in shear capacity than circular openings due to stress concentration effects at their sharp corners, resulting in a more brittle structural response. Mohamed et al. [12], through numerical analyses of deep beams with rectangular openings, recommended that openings should not interrupt the predicted compressive struts and that their depth should not exceed 20% of the overall beam depth. Furthermore, Ali et al. [13] demonstrated that both the opening aspect ratio and the reinforcement arrangement have a significant influence on strain distribution and the overall structural performance of deep beams.

Recent numerical and experimental investigations have provided further insight into the behavior of deep beams containing web openings. Advanced finite element analyses employing concrete damaged plasticity (CDP) models have confirmed that web openings significantly reduce stiffness and load-carrying capacity by interrupting the principal compressive stress trajectories within the beam. These studies consistently identify opening location as one of the most influential parameters governing structural performance, with the greatest reduction in strength occurring when the opening intersects the direct load-transfer path between the loading point and the support [14, 15]. Concerning opening geometry, several researchers have reported that square openings generally exhibit poorer structural performance than circular openings because of the severe stress concentrations developed at sharp corners [14, 15]. However, Al-Mahbashi et al. [16] suggested that when identical reinforcement interruption conditions are considered, the influence of opening shape on ultimate capacity may become less pronounced. Recent investigations have also indicated that increasing the opening-height-to-beam-depth ratio, particularly beyond approximately 0.33, may promote premature shear failure and alter the governing failure mechanism of deep beams. These findings collectively emphasize the importance of considering opening size, opening location, and reinforcement detailing when evaluating the structural response of deep beams containing web openings.

Despite the extensive body of research on deep beams with web openings, important gaps still remain in the available literature. Although several analytical and empirical approaches have been proposed, a reliable and widely accepted method for predicting the load-carrying capacity of deep beams with web openings is still lacking. Moreover, several recent numerical studies have relied on the assumption of perfect bond between internal components, thereby neglecting the potential influence of debonding mechanisms on structural response [15, 16]. In addition, many investigations have been based on relatively limited sample sizes and have focused primarily on static loading conditions. Although considerable attention has been devoted to comparing opening shapes and evaluating strengthening techniques, the combined influence of opening dimensions, vertical opening location, and reinforcement arrangement has not been systematically investigated. In particular, a need remains for detailed parametric studies addressing different vertical opening positions and their interaction with opening size in disturbed regions [14, 16]. The present study seeks to address this gap through a comprehensive finite element investigation of reinforced concrete deep beams containing square web openings of different sizes and vertical locations.

Accordingly, the present study investigates the nonlinear behavior of simply supported reinforced concrete deep beams containing web openings using the finite element method (FEM). A series of beams with identical overall dimensions subjected to concentrated loading were analyzed, incorporating openings of different sizes positioned at various vertical locations within the web. The primary variables considered in this study are opening size, opening location, and reinforcement arrangement around the opening region. Structural performance is evaluated in terms of ultimate load capacity, cracking load, crack propagation pattern, and principal stress distribution. The numerical model is first validated against available experimental results and subsequently employed in a comprehensive parametric investigation. Following the initial parametric investigation, a modified reinforcement arrangement was proposed around the openings, and its effectiveness in improving structural performance was evaluated.

Despite the considerable body of research on reinforced concrete deep beams with web openings, the combined influence of

square opening size, vertical opening location, and reinforcement detailing has not yet been systematically investigated within a unified numerical framework. Previous studies have generally focused on individual parameters, such as opening geometry, opening location, or strengthening techniques, without comprehensively examining their interaction and combined effect on load-transfer mechanisms within disturbed regions (D-regions).

To address this research gap, the present study provides a comprehensive finite element investigation of reinforced concrete deep beams containing square web openings. The scientific significance of this study lies in clarifying the combined influence of opening configuration and reinforcement detailing on crack development, principal stress redistribution, and structural performance. Furthermore, a practical reinforcement rearrangement strategy is proposed and evaluated to improve the structural behavior of deep beams with square web openings while requiring only a modest increase in reinforcement. The findings are expected to contribute to a better understanding of force-transfer mechanisms in disturbed regions and to provide practical guidance for the design of reinforced concrete deep beams containing service openings.

2. Finite element modeling and validation

2.1. Structural behavior of deep beams and numerical modeling approach

Reinforced concrete deep beams exhibit structural behavior significantly different from that of conventional slender beams due to their relatively small span-to-depth ratio. In such members, the assumption of a linear strain distribution across the section depth is no longer valid [17], and load transfer occurs primarily through compressive stress trajectories and discontinuity regions (D-regions) [2]. Consequently, conventional beam analysis methods based on classical flexural theory are unable to accurately represent the actual behavior of deep beams.

Several analytical approaches have been developed for the analysis of deep beams, including elasticity theory, the strut-and-tie model (STM), and the finite element method (FEM). Elasticity-based approaches are generally suitable for evaluating the pre-cracking behavior of deep beams; however, their applicability is limited in predicting the ultimate response of the member due to the assumption of linear material behavior. In contrast, the strut-and-tie model, which has been adopted by major design codes such as ACI 318, idealizes load transfer through an equivalent truss system composed of concrete compressive struts and steel tensile ties [17]. Although STM has proven to be an effective design tool [18–24], the determination of load paths and the configuration of struts largely depends on engineering judgment and may involve considerable uncertainty when applied to structural members with complex geometries, particularly those containing web openings.

The presence of openings in deep beams disrupts the natural load-transfer mechanism, alters or interrupts the concrete compressive struts, and generates significant stress concentrations around the opening boundaries. Under such circumstances, the finite element method (FEM) provides a more reliable framework for investigating the structural response due to its ability to simulate nonlinear material behavior, stress redistribution, crack initiation and propagation, and the interaction between concrete and reinforcing steel. Therefore, ABAQUS [25] was employed in the present study to perform nonlinear finite element analyses of reinforced concrete deep beams containing web openings.

2.2. Experimental specimens

To validate the accuracy of the developed finite element model, six reinforced concrete deep beams tested experimentally by El Maaddawy and Sherif [4] were numerically simulated using ABAQUS [25]. The numerical results obtained from the present study were subsequently compared with both the experimental data and the finite element results reported by Hawileh et al. [6].

The investigated specimens had overall dimensions of 1200 mm × 500 mm × 80 mm and contained square web openings measuring either 200 mm × 200 mm or 250 mm × 250 mm. The openings were positioned at different locations within the shear span to produce varying degrees of disturbance in the load-transfer mechanism and the concrete compressive struts. The geometric characteristics, opening configurations, and specimen designations are summarized in Table 1 and illustrated in Fig. 1.

For all specimens, the concrete compressive strength was taken as 21 MPa. Longitudinal reinforcement and web reinforcement were modeled using S420 and S300 steel grades, respectively. Detailed reinforcement layouts of the tested beams are illustrated in Fig. 2.

Table 1. Characteristics of the studied deep beam specimens.

Specimen	Opening location	Opening size (mm)
NS-200-T	Top of shear span	200 × 200
NS-250-T	Top of shear span	250 × 250
NS-200-C	Middle of shear span	200 × 200
NS-250-C	Middle of shear span	250 × 250
NS-200-B	Bottom of shear span	200 × 200
NS-250-B	Bottom of shear span	250 × 250

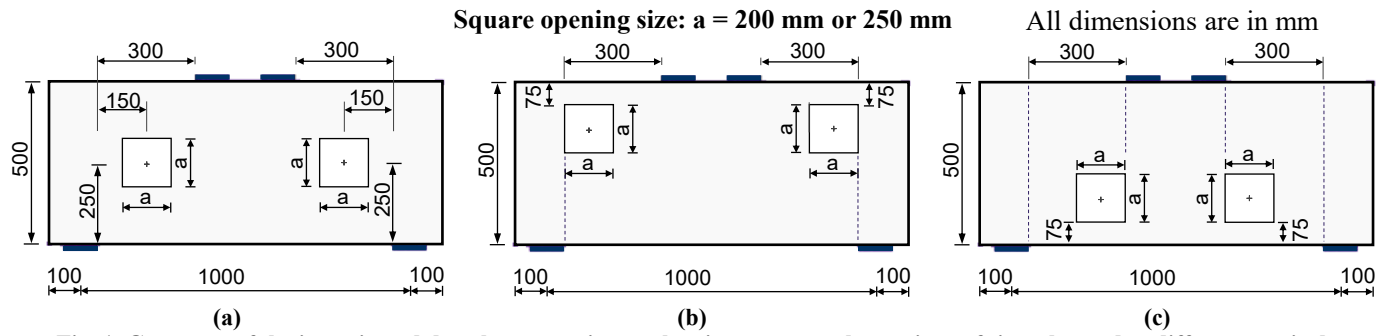


Fig. 1. Geometry of the investigated deep beam specimens showing square web openings of size a located at different vertical positions within the shear span: (a) center; (b) top; and (c) bottom [4].

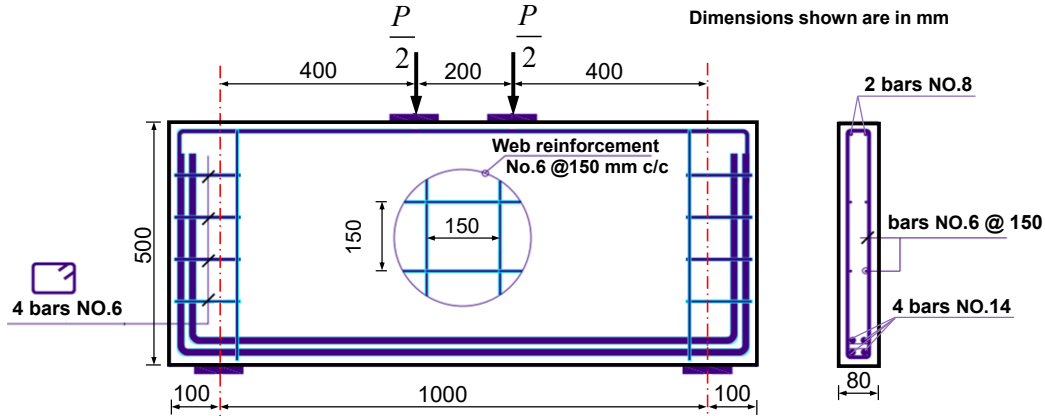


Fig. 2. Geometrical dimensions, reinforcement layout, loading arrangement, and cross-sectional details of the experimentally tested deep beam specimen used for model validation [4].

2.3. Material constitutive models

2.3.1. Concrete behavior under uniaxial compression and tension

To simulate the nonlinear compressive behavior of concrete, the Hognestad stress-strain model [26] was adopted. The Hognestad model was selected because it provides a realistic representation of the nonlinear compressive behavior of normal-strength concrete while maintaining a relatively simple mathematical formulation. Owing to its proven accuracy and extensive use in numerical studies of reinforced concrete structures, it is considered an appropriate constitutive model for the present investigation. This model consists of a parabolic ascending branch up to the peak compressive strength, a peak point corresponding to a compressive strain of 0.002, and a descending softening branch in which the stress gradually decreases with increasing strain. In the present study, the ultimate concrete strain was taken as 0.0038, and the post-peak response was represented by a linear descending branch.

For modeling the tensile behavior of concrete, a simplified stress-strain relationship was employed. In this model, the tensile stress increases linearly up to the tensile strength of concrete, followed by a sudden stress drop to 60% of the peak tensile strength. Thereafter, the stress decreases linearly until it reaches zero. The compressive and tensile constitutive relationships adopted for concrete are illustrated in Fig. 3 [6].

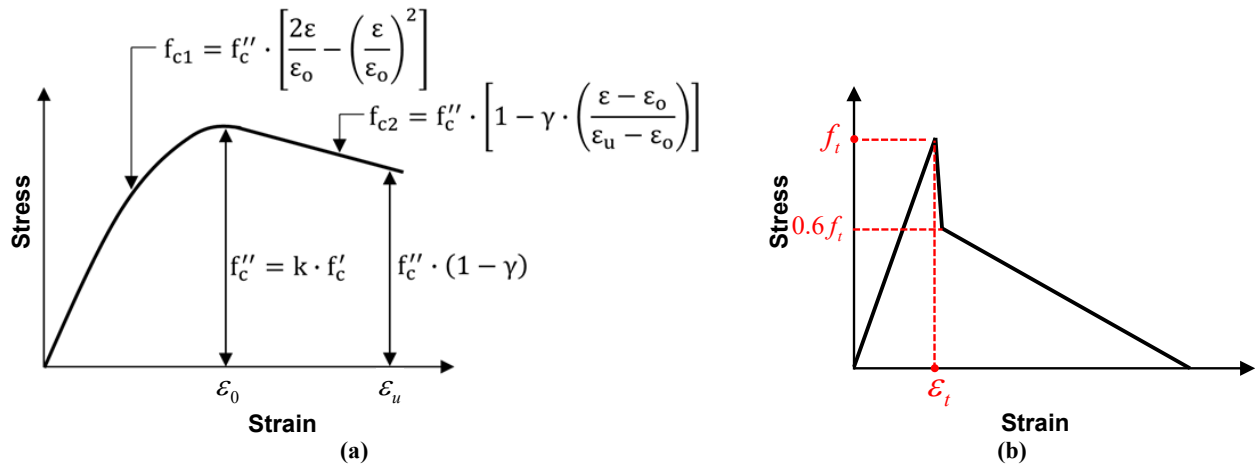


Fig. 3. Concrete stress-strain relationships adopted in the numerical model: (a) uniaxial compressive; and (b) Uniaxial Tensile.

2.3.2. Reinforcing steel constitutive model

The reinforcing steel was modeled using a bilinear elastic-plastic constitutive law incorporating strain hardening. This constitutive model was adopted because it accurately reproduces the yielding and post-yield hardening behavior of conventional

reinforcing bars while remaining computationally efficient and numerically stable in nonlinear finite element analyses. The initial portion of the curve represents linear elastic behavior, while beyond the yield point the material response continues with a strain-hardening slope equal to 2% of the initial elastic modulus [27]. The modulus of elasticity and Poisson's ratio of the reinforcing steel were assumed to be 200 GPa and 0.30, respectively [2]. The stress-strain relationships adopted for S300 and S420 reinforcing steels are presented in Fig. 4.

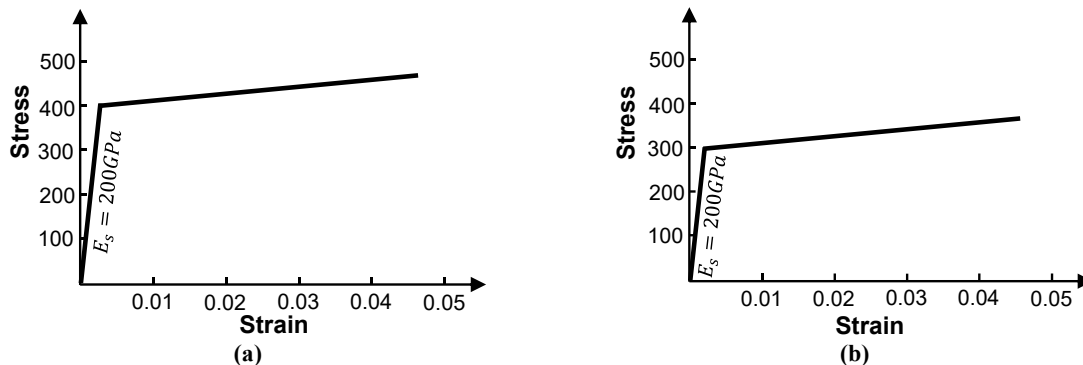


Fig. 4. Stress-strain relationships adopted for reinforcing steel modeling: (a) grade 420 steel; and (b) grade 300 steel.

2.3.3. Concrete damaged plasticity (CDP) model

To capture the multiaxial behavior of concrete, the concrete damaged plasticity (CDP) model available in ABAQUS [25] was employed. This constitutive model combines the principles of plasticity theory and continuum damage mechanics, enabling the simulation of tensile cracking, compressive crushing, progressive stiffness degradation, and post-peak material softening within a unified framework.

Compared with alternative constitutive models available in ABAQUS [25], the concrete damaged plasticity (CDP) model provides a comprehensive framework for simulating the nonlinear behavior of reinforced concrete under complex multiaxial stress states. The model is capable of representing tensile cracking, compressive crushing, stiffness degradation, damage evolution, and the distinct responses of concrete under tension and compression through independent damage variables. These capabilities are particularly important for the present study, in which the presence of square web openings results in severe stress redistribution, localized cracking, and progressive deterioration of the load-transfer mechanism within disturbed regions (D-regions). Furthermore, the CDP model has been extensively validated and widely adopted in numerical investigations of reinforced concrete structures, demonstrating both reliable predictive capability and numerical robustness. For these reasons, the CDP model was selected as the constitutive model for concrete throughout the present finite element analyses.

2.4. Finite element model

2.4.1. Element types

The numerical simulations were performed using a three-dimensional finite element model developed in ABAQUS [25]. Concrete was modeled using eight-node linear brick elements with reduced integration (C3D8R). These elements have been widely employed in nonlinear analyses of reinforced concrete structures due to their computational efficiency and their ability to alleviate shear locking problems while maintaining satisfactory accuracy.

The reinforcing bars were modeled using two-node truss elements (T3D2), which are capable of transmitting only axial forces and therefore provide an appropriate representation of the mechanical behavior of reinforcing steel embedded in concrete members.

2.4.2. Boundary conditions and loading configuration

The validated beam specimens were modeled as simply supported members with an effective span of 1000 mm., a width of 80 mm, and a depth of 500 mm, with an effective span of 1000 mm. All specimens were subjected to four-point bending loading conditions. The beams were modeled as simply supported members, and two concentrated loads were applied symmetrically on the top surface of the beam at a distance of 400 mm from each support.

This loading arrangement generated significant shear stresses within the shear spans, where the web openings were located, thereby providing suitable conditions for investigating the influence of openings on the load-transfer mechanism, stress distribution, and structural performance of reinforced concrete deep beams.

A sufficiently refined finite element mesh was adopted to accurately capture the stress concentration around the web openings while maintaining computational efficiency. The nonlinear analyses were performed using the ABAQUS [25] implicit solver with automatic load incrementation until convergence was achieved at each increment.

2.5. Model validation

2.5.1. Load-displacement response

To assess the accuracy of the developed finite element model, the load-displacement responses obtained from the numerical

simulations were compared with the experimental results reported by El Maaddawy and Sherif [4], as well as the numerical results presented by Hawileh et al. [6]. Fig. 5 illustrates the comparison of the load-displacement curves for the investigated specimens.

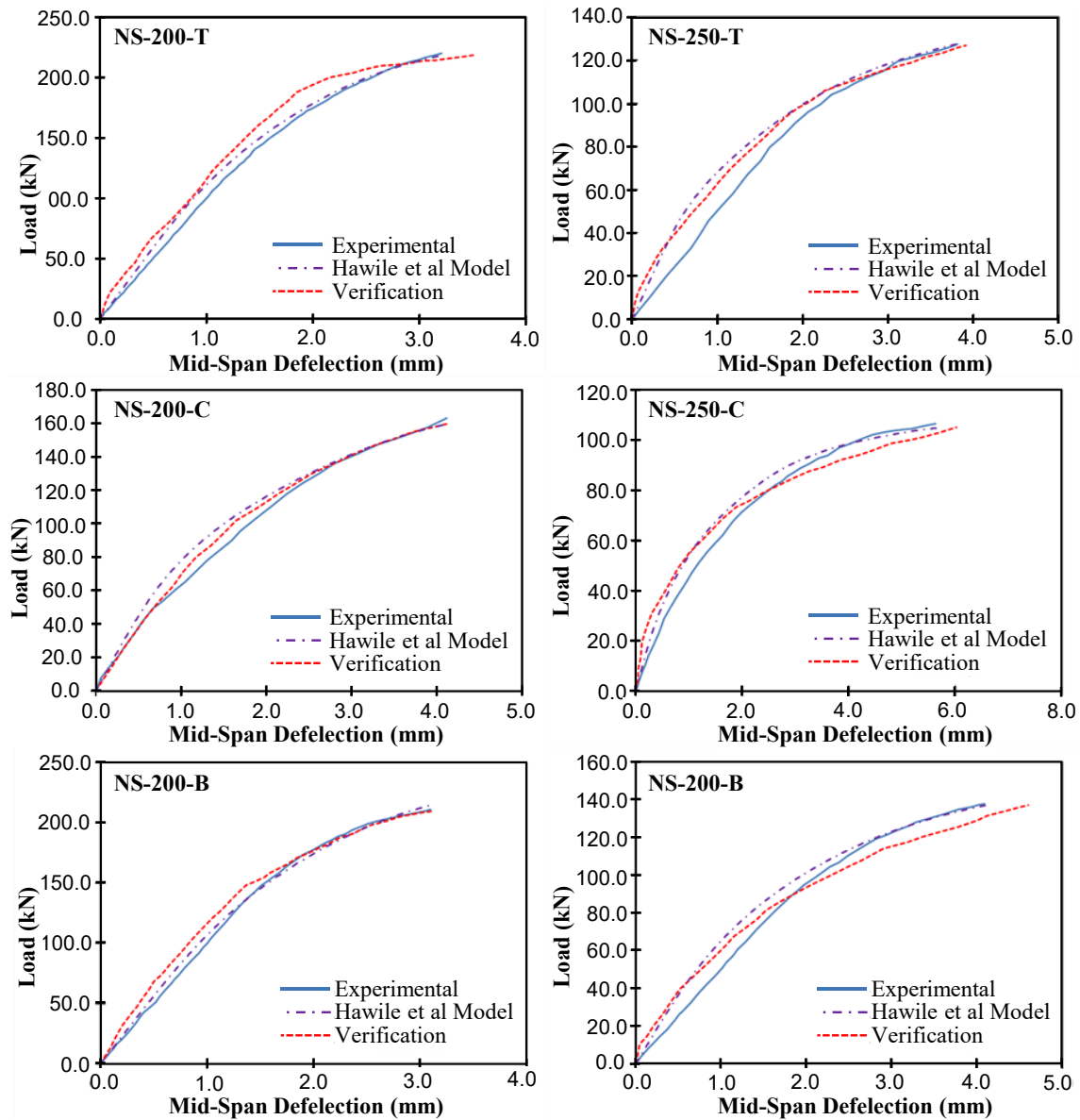


Fig. 5. Comparison of midspan load-displacement curves for the validation specimens obtained from experimental tests, previous numerical studies, and the present FE model.

The results demonstrate that the proposed model is capable of accurately reproducing the initial stiffness, nonlinear response, cracking load, and ultimate load-carrying capacity of the deep beams. As illustrated in Fig. 5, the numerical load-displacement curves show good agreement with the experimental responses throughout the loading history. This visual agreement, together with the close correspondence of the crack propagation patterns and the quantitative comparison of the P_{cr} / P_u , confirms the reliability of the developed finite element model for simulating the structural behavior of reinforced concrete deep beams containing web openings.

2.5.2. Crack patterns and failure mechanisms

To evaluate the capability of the numerical model in predicting crack initiation and propagation, the crack patterns obtained from the finite element analyses were compared with the corresponding experimental observations and the numerical results reported by Hawileh et al. [6]. The comparisons for specimens NS-200-T, NS-200-B, and NS-250-C are presented in Fig. 6. The numerical model accurately predicted both the crack initiation locations and the principal crack propagation paths observed in the experimental specimens. In all investigated beams, diagonal cracks originated from the corners of the square openings and propagated toward the loading and support regions, demonstrating a high level of agreement with the experimental observations.

2.5.3. Quantitative validation based on the cracking load ratio (P_{cr} / P_u)

To further evaluate the predictive capability of the developed finite element model, a quantitative validation was performed using the ratio of cracking load to ultimate load (P_{cr} / P_u). This parameter provides a meaningful indicator of the onset of cracking relative to the ultimate structural capacity and reflects the ability of the numerical model to capture the transition from the elastic stage to the nonlinear response of reinforced concrete deep beams.

The experimental results reported by El Maaddawy and Sherif [4] indicate that the cracking load occurred at approximately 31–51% of the ultimate load capacity for the investigated specimens. The corresponding values predicted by the present finite element model ranged from 27% to 52%. The close overlap between these ranges demonstrates that the numerical model provides a reliable prediction of crack initiation and the subsequent stiffness degradation associated with the development of diagonal cracking.

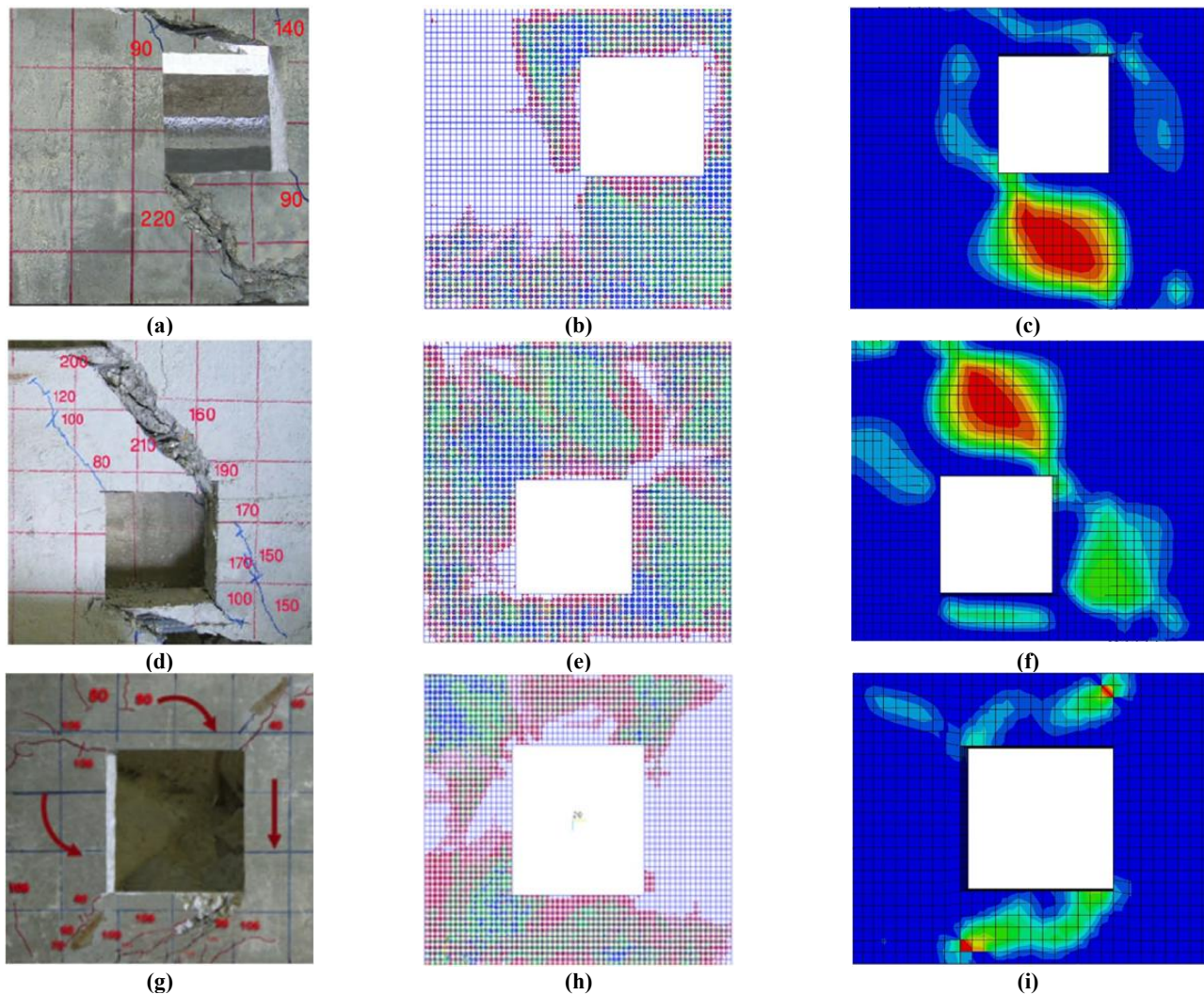


Fig. 6. Comparison of experimentally observed and numerically predicted crack patterns for validation specimens NS-250-C, NS-200-B, and NS-200-T: (a, d, and g) experimental results; (b, e, and h) reference numerical results; and (c, f, and i) predictions of the present finite element model.

3. Results and discussion

3.1. Studied specimens

To investigate the influence of square web openings on the structural behavior of reinforced concrete deep beams, six numerical specimens were analyzed using nonlinear finite element analysis (FEA). The specimens were categorized according to two geometric parameters: (I) opening size (200 mm and 250 mm) and (II) vertical opening location within the web (center, top, and bottom). These opening locations were intentionally selected to produce different levels of interference with the principal compression strut that transfers load from the loading point to the support.

It should be noted that the geometrical configuration, material properties, and reinforcement details of the investigated specimens are identical to those used in the validation study presented in Section 2. Therefore, the validated numerical models were adopted as the reference specimens in the present parametric investigation. To establish a consistent and scalable nomenclature suitable for future studies involving different opening geometries, a new specimen designation system was introduced. In this notation, SO denotes a square opening, the following number indicates the opening size (mm), and the final letter identifies the opening location within the beam depth (C = Center, T = Top, and B = Bottom). Furthermore, specimens with modified reinforcement arrangements are identified by the suffix –M. The selected specimens represent all possible combinations of the investigated opening sizes and vertical opening locations considered in this study.

Accordingly, the six reference specimens investigated in this study are designated as SO-200C, SO-200T, SO-200B, SO-250C, SO-250T, and SO-250B. The corresponding specimens with modified reinforcement layouts are denoted as SO-200C-M, SO-200T-M, SO-200B-M, SO-250C-M, SO-250T-M, and SO-250B-M, respectively.

The parametric investigation focuses on evaluating the effects of opening size and vertical opening location on the structural

response of deep beams and assessing the effectiveness of a modified reinforcement arrangement around the openings. In addition, a reinforcement modification strategy is proposed and assessed to mitigate the adverse effects of geometric discontinuities introduced by the openings. The effectiveness of the proposed reinforcement arrangement is subsequently evaluated through comparisons of load-displacement responses, cracking patterns, and stress-field distributions.

3.2. Crack patterns and failure mechanisms

Fig. 7 illustrates the initiation and propagation patterns of cracks in deep beams with web openings obtained from the finite element analyses, presented in the form of tensile plastic strain contours. Examination of the results indicates that both the crack initiation locations and propagation paths are strongly influenced by the size and vertical position of the openings. In all specimens, the corners of the square openings acted as critical regions for the initiation of diagonal shear cracks due to geometric discontinuities and the associated stress concentrations.

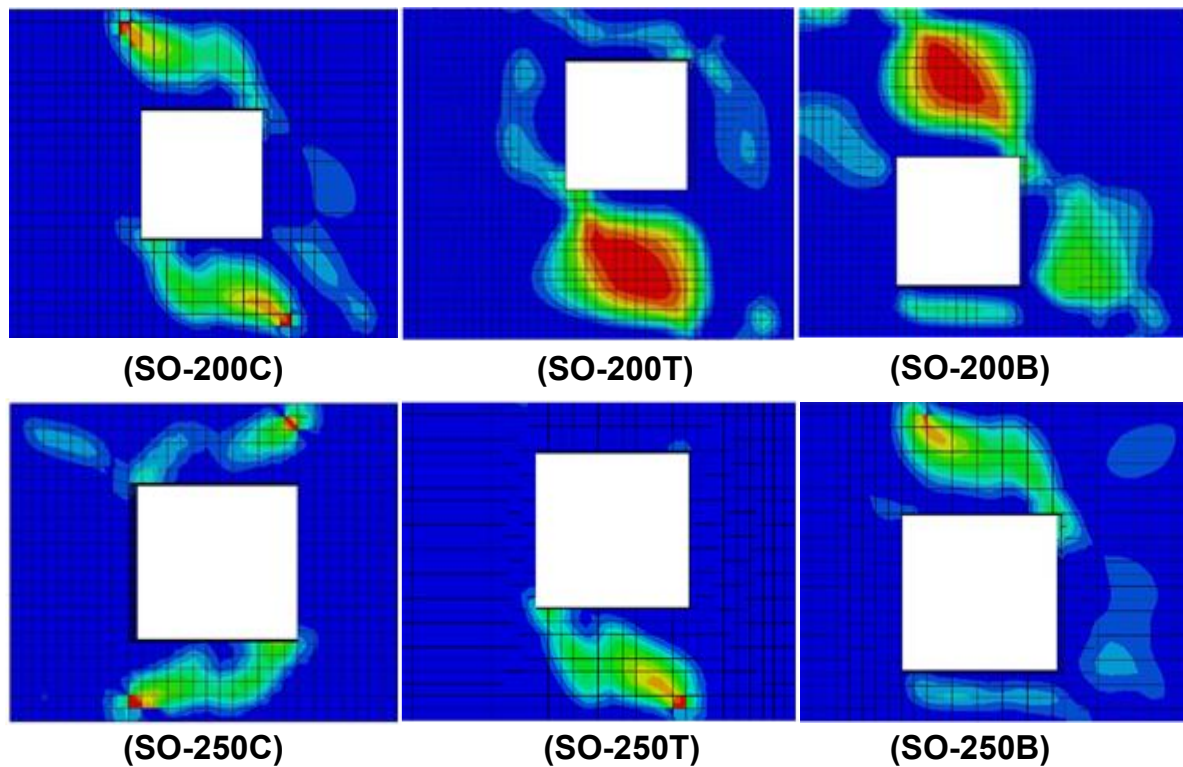


Fig. 7. Comparison of crack propagation patterns in deep beams with square web openings of different sizes and vertical positions.

In specimen SO-200T, the first diagonal crack initiated at the lower corner of the opening and propagated toward the support along the principal load-transfer path. As the applied load increased, another crack developed from the upper corner of the opening toward the loading point, eventually leading to the formation of a shear failure mechanism within the beam web. In contrast, for Specimen SO-200B, where the opening was located in the lower portion of the web, the initial crack propagated from the upper corner of the opening toward the loading region, followed by the development of a second crack extending from the lower corner toward the support. This difference in crack patterns highlights the direct dependence of the failure mechanism on the location of the opening relative to the principal concrete compression strut.

For Specimen SO-200C, in which the opening was positioned at the mid-depth of the web, diagonal cracks initiated almost simultaneously from opposite corners of the opening. This behavior suggests that locating the opening at the center of the web disrupts the principal compression field above and below the opening simultaneously, resulting in the most severe disturbance to the internal stress flow. Furthermore, increasing the opening size from 200 mm to 250 mm led to a noticeable increase in both the width and extent of the diagonal cracks. This observation indicates intensified stress concentration effects and a reduction in the effective concrete area available for force transfer.

3.3. Principal stress distribution

To provide further insight into the mechanisms governing the behavior of the investigated deep beams, the contours of the minimum principal stresses (compressive stresses) and maximum principal stresses (tensile stresses) are presented in Figs. 8 and 9, and Figs. 10 and 11, respectively.

Examination of the minimum principal stress contours indicates that the presence of an opening alters the natural load-transfer path of compressive stresses and causes stress concentrations around the opening. This phenomenon is particularly pronounced in Specimens SO-200C and SO-250C, where the opening is located at the center of the shear span. In these specimens, the opening interrupts the direct path of the principal compression strut extending between the loading point and the support, forcing the compressive stresses to be transferred through the narrow concrete regions above and below the opening. Consequently, stress concentrations increase, leading to a reduction in the load-carrying capacity of the member. In contrast, for Specimen SO-200T, a

substantial portion of the compressive stress field bypasses the opening by passing beneath it and is subsequently transferred to the support. This behavior explains the higher load-carrying capacity observed in this specimen.

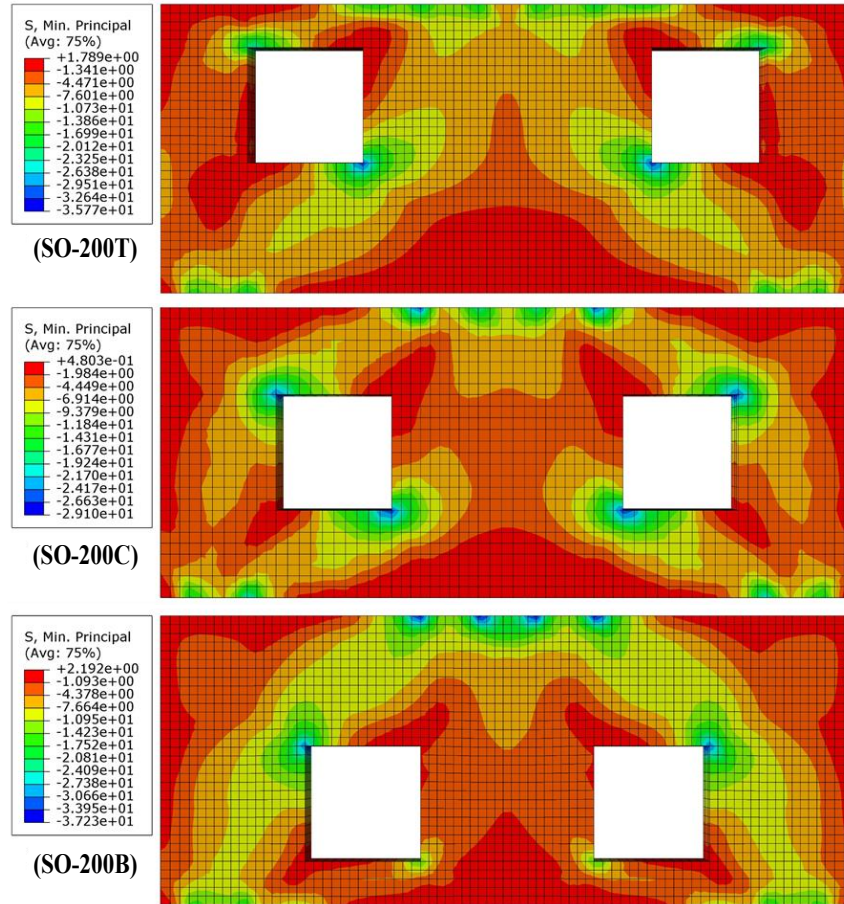


Fig. 8. Comparison of minimum principal stress fields in deep beam specimens with 200×200 mm web openings.

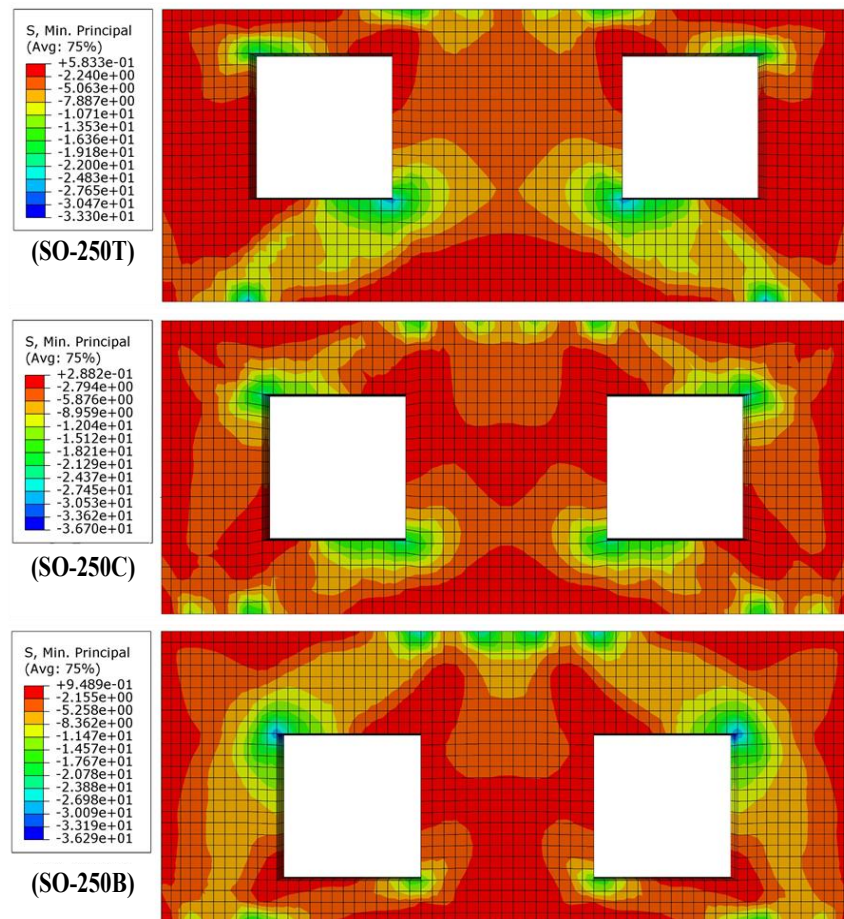


Fig. 9. Comparison of minimum principal stress fields in deep beam specimens with 250×250 mm web openings.

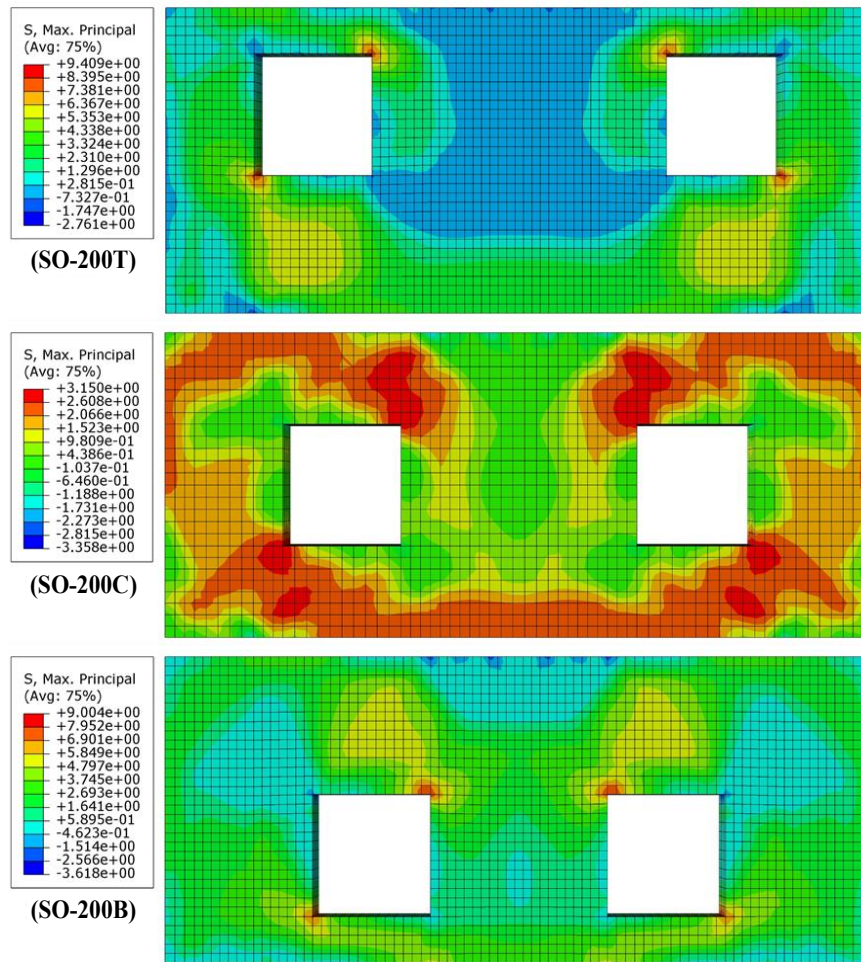


Fig. 10. Comparison of maximum principal stress fields in deep beam specimens with 200×200 mm web openings.

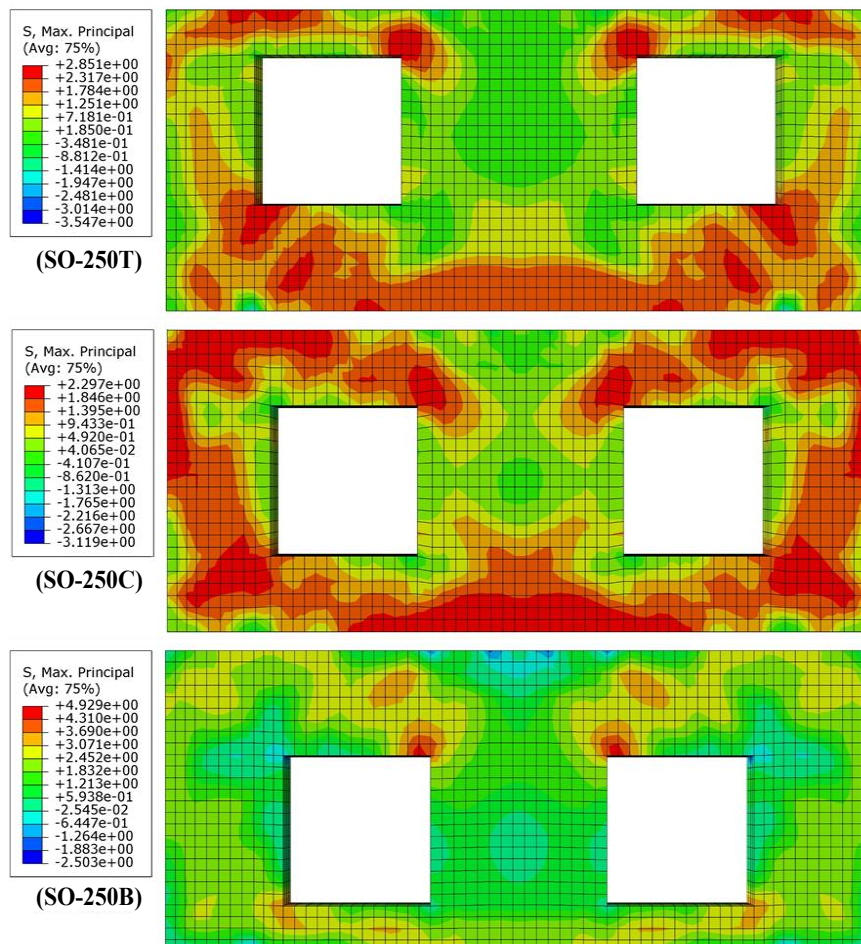


Fig. 11. Comparison of maximum principal stress fields in deep beam specimens with 250×250 mm web openings.

The maximum principal stress contours further reveal that the highest tensile stresses are predominantly concentrated at the corners of the openings, which correspond to the initiation points of the diagonal cracks. Although variations in the size and location of the openings do not significantly alter the overall pattern of stress distribution, they have a substantial influence on the magnitude of local stress concentrations. Within the group containing 200-mm openings, Specimen SO-200T exhibited the highest localized tensile stress, exceeding those of Specimens SO-200C and SO-200B by 198.7% and 4.5%, respectively. Similarly, within the group containing 250-mm openings, Specimen SO-250B experienced the highest tensile stress, with values 114% and 72% greater than those recorded in Specimens SO-250C and SO-250T, respectively. These findings indicate that increasing the opening size becomes particularly detrimental when the opening is located close to the tension zone of the beam, where stress concentrations and crack development become more pronounced.

3.4. Effect of opening location

To isolate the influence of opening location, specimens with identical opening sizes were compared. For both opening sizes (200 mm and 250 mm), centrally located openings produced the greatest deterioration in structural performance because they directly interrupted the principal compression strut connecting the loading point to the support. Conversely, openings located near the top of the web caused the least disturbance to the load-transfer mechanism, while bottom openings exhibited intermediate behavior. This trend was consistently observed in terms of crack propagation, principal stress redistribution, and ultimate load capacity.

3.5. Effect of opening size

To evaluate the influence of opening size independently, specimens with identical opening locations were compared. Increasing the opening size from 200 mm to 250 mm consistently intensified stress concentrations, enlarged the disturbed region, and reduced the effective concrete area available for force transfer. Consequently, larger openings resulted in wider diagonal cracks, greater disruption of the principal compression strut, and lower structural capacity, regardless of the opening location.

3.6. Proposed reinforcement arrangement

The results presented in the preceding sections demonstrated that the presence of a square opening leads to stress concentration at the opening corners, disruption of the principal compression strut, and accelerated propagation of diagonal cracks. Therefore, to improve the structural performance of deep beams without modifying the member geometry or the material properties, a modified web reinforcement arrangement was proposed. The proposed reinforcement arrangement was developed based on the observed stress distribution and crack propagation obtained from the validated finite element analyses. Rather than increasing the total reinforcement ratio uniformly, the modification focused on redistributing the existing web reinforcement toward regions where the principal compression strut was interrupted and stress concentrations were most severe. Additional confinement reinforcement was introduced around the opening corners to improve concrete confinement, restrict diagonal crack propagation, and facilitate stress redistribution within the disturbed region (D-region). Consequently, the proposed detailing aims to restore a more continuous load-transfer mechanism while requiring only a modest increase in the total amount of reinforcing steel. The primary objective of this strategy was to strengthen the load-transfer paths around the opening, enhance concrete confinement within critical regions, and mitigate stress concentration and crack propagation in the vicinity of the opening.

For specimens with centrally located openings (specimens SO-200C and SO-250C), where the greatest disruption of the principal compression strut was observed, two longitudinal web reinforcement bars were relocated to the regions immediately above and below the opening. These bars were intended to act as localized tie elements and redistribute a portion of the forces interrupted by the opening. In addition, supplementary stirrups were provided along both sides and the longitudinal extent of the opening to improve concrete confinement and enhance the stability of the compressive stress fields.

For specimens with openings located in the upper portion of the web (specimens SO-200T and SO-250T), the proximity of the opening to the upper compression reinforcement reduced the need for extensive reinforcement modifications above the opening. In this case, the web reinforcement bars interrupted by the opening were relocated beneath it, and an additional longitudinal reinforcement bar was placed between the opening and the bottom face of the beam. Furthermore, supplementary stirrups were installed around the opening to control stress concentrations and improve force transfer.

For specimens with openings located in the lower portion of the web (specimens SO-200B and SO-250B), the arrangement of the primary bottom flexural reinforcement was maintained unchanged due to the proximity of the opening to the main tension reinforcement. Instead, the web reinforcement bars interrupted by the opening were relocated above it, and an additional longitudinal reinforcement bar was placed between the opening and the top face of the beam within the loading region. Additional stirrups were also provided around the opening to enhance the performance of the discontinuity region (D-region) and to control crack development.

For all specimens, a clear distance of 10 mm, equal to the original concrete cover, was maintained between the reinforcement and the opening edges. This measure ensured constructability while preventing excessive reinforcement congestion around the opening. It should be noted that implementation of the proposed reinforcement scheme resulted in only a limited increase in the amount of reinforcing steel, ranging from approximately 4% to 8.4%. Therefore, the proposed solution can be considered both practical and economically viable. Fig. 12 presents the detailed reinforcement layouts adopted for the different specimens. The specimens incorporating the modified reinforcement arrangement are designated by the suffix “-M”; for example, SO-200C-M denotes the modified version of specimen SO-200C.

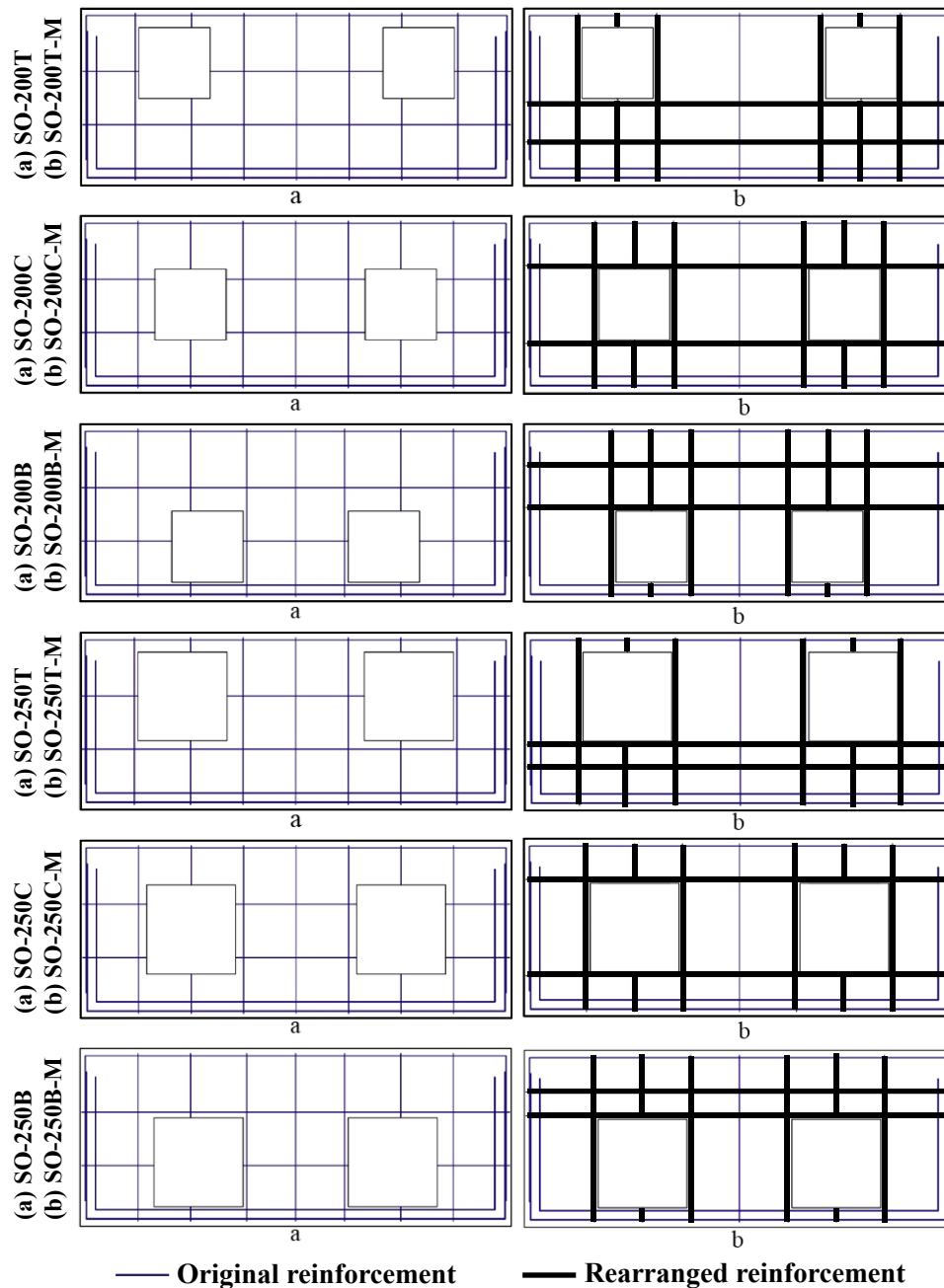


Fig. 12. Comparison of: (a) the original reinforcement layout; and (b) the modified reinforcement layout proposed for deep beam specimens with square web openings.

3.6.1. Effect on load-displacement response

To evaluate the effectiveness of the proposed reinforcement arrangement, nonlinear finite element analyses were repeated for all modified specimens, and the resulting midspan load-displacement curves were compared with those of the original specimens. Fig. 13 presents a comparison of the load-displacement responses before and after the reinforcement modification.

Examination of these curves indicates that the proposed reinforcement arrangement had a significant influence on the overall behavior of the deep beams with web openings. Although the initial portions of the curves, corresponding to the elastic response of the members, exhibit only minor differences in most specimens, the effect of the reinforcement modification becomes clearly evident after crack initiation. In all specimens, the redistribution of web reinforcement resulted in an increase in the cracking load, improved stability of the load-carrying mechanism, and enhanced ultimate load-carrying capacity.

For the specimens with 200-mm openings, the ultimate load-carrying capacity increased by approximately 3% to 9%. Moreover, the load-displacement curves indicate that, following the formation of the initial cracks, the reduction in structural stiffness occurred at a slower rate in the modified specimens. Consequently, these specimens were capable of sustaining higher loads at comparable displacement levels. This behavior suggests that the reinforcement relocated around the openings effectively contributed to partially restoring the load-transfer mechanism interrupted by the presence of the openings.

A similar trend was observed for the specimens with 250-mm openings. However, the influence of the reinforcement modification was more pronounced in this group, resulting in increases in ultimate load-carrying capacity ranging from 5% to 10%. This finding indicates that as the opening size increases and the disturbance to the internal stress field becomes more severe, the

contribution of the additional reinforcement to force redistribution and control of the discontinuity region becomes increasingly important. In other words, the greater the reduction in the effective web area caused by the opening, the more effective reinforcement-detailing strategies become in mitigating the associated loss of structural performance.

As shown in Fig. 13 further reveals that the specimens with centrally located openings experienced the greatest improvement. This behavior can be attributed to the fact that, in these specimens, the opening causes the most severe interruption of the principal compression strut. Consequently, the additional reinforcement placed around the opening is able to play a more effective role in establishing alternative load-transfer paths. In contrast, the specimens with upper openings, which initially caused the least disturbance to the principal compression strut, exhibited relatively smaller improvements following the reinforcement modification.

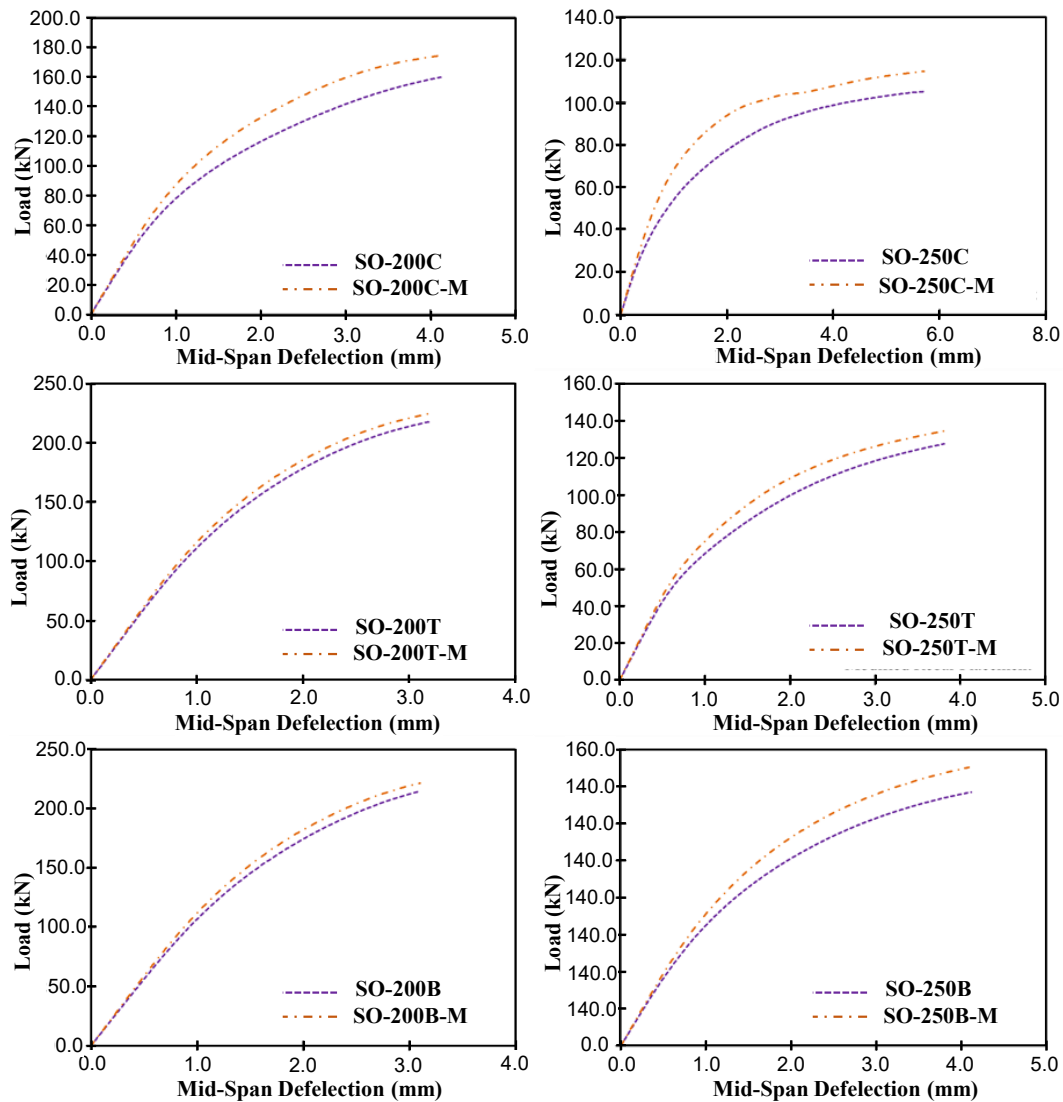


Fig. 13. Comparison of midspan load-displacement behavior of deep beam specimens before and after reinforcement layout modification.

Overall, the load-displacement responses demonstrate that the strategic concentration of web reinforcement in the vicinity of the openings not only enhances the ultimate load-carrying capacity but also improves the nonlinear behavior and structural stability of deep beams by delaying damage initiation and reducing the rate of stiffness degradation after cracking. These findings confirm that reinforcement rearrangement can serve as a practical and cost-effective strengthening strategy for compensating a portion of the performance degradation associated with the introduction of openings in reinforced concrete deep beams.

3.6.2. Effect on crack development and cracking load

The crack patterns and contours of maximum tensile plastic strains for the modified specimens are presented in Fig. 14. Comparison of the results before and after reinforcement rearrangement indicates that relocating the web reinforcement and providing additional stirrups around the openings effectively improved the stress transfer mechanism within the disturbed regions (D-regions) and reduced local stress concentrations at critical locations.

The beneficial effect of the proposed reinforcement arrangement was particularly evident in specimens SO-250C-M and SO-250B-M, which contained the larger square openings located at the central and lower levels of the web, respectively. In these specimens, the additional reinforcement surrounding the opening enhanced concrete confinement and promoted a more uniform distribution of internal forces around the discontinuity. As a result, the tensile plastic strains previously concentrated at the opening corners were significantly reduced.

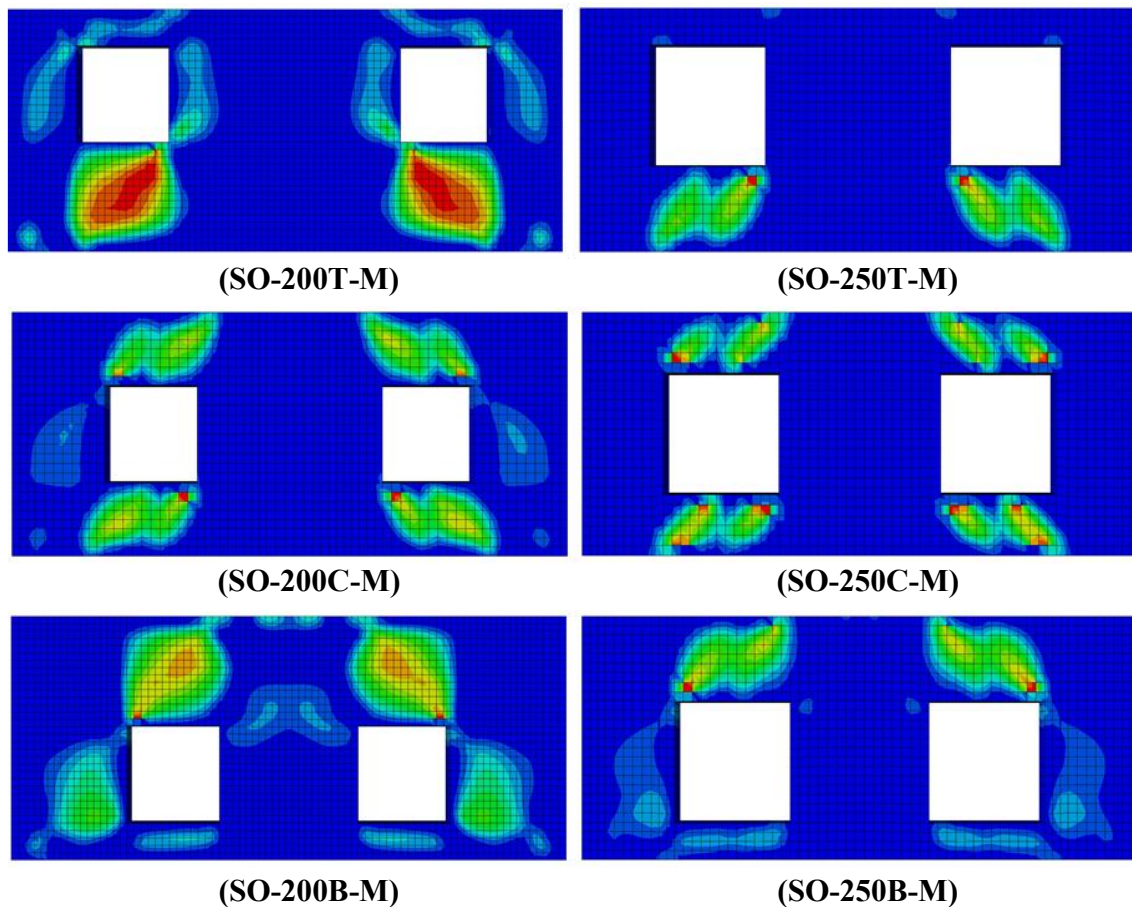


Fig. 14. Crack patterns of reinforced concrete deep beams with square web openings after reinforcement rearrangement.

A notable observation is that the stirrups placed adjacent to the opening restricted the outward propagation of tensile plastic strain zones originating from the upper corners of the opening. Consequently, crack development became more localized and controlled, indicating an improvement in the structural behavior of the discontinuity region. Although the overall crack trajectories remained generally similar to those observed in the original specimens, the intensity and extent of critical cracking were noticeably reduced after reinforcement modification.

In addition to crack control, the proposed reinforcement arrangement improved the nonlinear deformation capacity of the beams. The maximum tensile plastic strain increased by 16.4%, 2.67%, and 48.8% in specimens SO-200C-M, SO-200T-M, and SO-200B-M, respectively, compared with their corresponding reference specimens. These increases indicate a greater ability of the modified specimens to redistribute stresses and sustain inelastic deformations prior to failure. The increase in tensile plastic strain capacity suggests that the modified specimens were able to sustain larger inelastic deformations prior to failure.

Overall, the results demonstrate that although reinforcement rearrangement does not fundamentally alter the governing cracking mechanism, it effectively mitigates stress concentration around the opening, improves concrete confinement, and enhances the nonlinear response of deep beams. These findings highlight the importance of reinforcement detailing in the vicinity of web openings and confirm that an appropriate redistribution of web reinforcement can significantly improve the cracking performance and overall structural behavior of reinforced concrete deep beams with square openings.

A quantitative assessment of the effectiveness of the proposed reinforcement arrangement was performed by comparing the results of the modified specimens presented in Table 2 with those of the original specimens. The comparison indicates that the reinforcement modification successfully delayed the initiation of the first diagonal shear crack in all specimens. Depending on the opening size and location, the cracking load increased by 4.5% to 44%, demonstrating the effectiveness of the proposed detailing in improving the cracking resistance of deep beams with web openings.

The smallest improvement was observed in specimens with openings located near the upper part of the web (specimens SO-200T-M and SO-250T-M). In these specimens, the openings caused relatively limited interference with the main compression strut, and therefore the potential for further improvement was inherently restricted. In contrast, the largest enhancement occurred in specimens with centrally located openings (specimens SO-200C-M and SO-250C-M), where the opening interrupted the primary load-transfer path most severely. In such cases, the relocated reinforcement effectively restored part of the disrupted force-transfer mechanism, resulting in a substantial increase in cracking load.

The results further show that the (P_{cr} / P_u) ratio increased in all modified specimens, with improvements ranging from 1% to 33%. Since this ratio reflects the relative delay in the onset of nonlinear behavior, the observed increase indicates that the proposed reinforcement arrangement enhanced the structural stability of the beams and postponed crack propagation under increasing load levels.

The influence of opening size was also found to be significant. As the opening dimension increased from 200 mm to 250 mm, the effectiveness of the reinforcement modification became more pronounced. For example, the increase in cracking load achieved in the specimens with 250-mm openings was substantially greater than that obtained for the corresponding specimens with 200-mm openings. This trend suggests that the structural benefits of reinforcement redistribution become increasingly important as the severity of the geometric discontinuity increases. Therefore, appropriate reinforcement detailing around large web openings can play a critical role in controlling stress concentration and improving force redistribution within disturbed regions (D-regions).

Table 2. Comparison of cracking and ultimate loads of deep beams with openings before and after reinforcement layout modification.

Specimen	Original reinforcement layout			Modified reinforcement layout			Increase in cracking load (%)
	Cracking load (kN)	Ultimate load (kN)	P_{cr} / P_u	Cracking load (kN)	Ultimate load (kN)	P_{cr} / P_u	
SO-200C	43.16	159.64	0.27	62.31	174.21	0.36	44.38
SO-200T	75.55	208.08	0.35	78.99	224.83	0.35	4.55
SO-200B	81.62	214.53	0.38	91.95	221.39	0.41	12.65
SO-250C	41.93	104.94	0.40	52.02	114.44	0.46	24.06
SO-250T	42.86	127.54	0.34	48.92	134.51	0.36	14.13
SO-250B	36.16	136.76	0.26	47.60	150.37	0.32	31.6

3.6.3. Effect on minimum principal stress distribution

To evaluate the influence of reinforcement rearrangement on the load-transfer mechanism and the performance of the discontinuity region surrounding the opening, the contours of the minimum principal stresses (compressive stresses) after the reinforcement modifications are presented in Figs. 15 and 16 for specimens with 200-mm and 250-mm openings, respectively. Comparison of these contours with those obtained from the original specimens indicates that relocating a portion of the web reinforcement around the opening and increasing the reinforcement density within critical regions resulted in a more uniform distribution of compressive stresses and a reduction in stress concentrations at the opening corners. Consequently, the load-transfer paths around the opening became more organized, and the disturbance introduced into the compressive stress field was partially mitigated.

The quantitative results further support these observations. For the specimens with 200-mm openings, the magnitude of the minimum principal stresses in the vicinity of the opening increased by 57.6%, 14.5%, and 60.2% for specimens SO-200C-M, SO-200T-M, and SO-200B-M, respectively. Similarly, for the specimens with 250-mm openings, the corresponding increases for specimens SO-250C-M, SO-250T-M, and SO-250B-M were 40.3%, 37.5%, and 29.5%, respectively. These results indicate that the effectiveness of the reinforcement modification was more pronounced in specimens with openings located in the central or lower regions of the web. In these cases, the openings caused more severe interference with the principal compression strut, allowing the reinforcement rearrangement to play a more significant role in stress redistribution.

A comparison between the two opening groups further reveals that, although the reinforcement modification improved the distribution of compressive stresses in all specimens, the magnitude of this improvement decreased in some cases as the opening size increased from 200 mm to 250 mm. This behavior suggests that as the geometric discontinuity becomes larger, the disturbance to the load-transfer mechanism intensifies, thereby limiting the ability of the additional reinforcement to redistribute stresses effectively.

Overall, the results presented in Figs. 15 and 16 demonstrate that the proposed reinforcement arrangement successfully improved the structural behavior of deep beams with square openings by reducing compressive stress concentrations around the openings and enhancing the performance of the concrete compression struts. These findings highlight the importance of reinforcement detailing within discontinuity regions (D-regions) and emphasize its significant role in improving the nonlinear response of reinforced concrete deep beams.

3.6.4. Effect on maximum principal stress distribution

To assess the influence of reinforcement rearrangement on the tensile stresses governing crack initiation and propagation in concrete, the contours of the maximum principal stresses after the reinforcement modifications are presented in Figs. 17 and 18 for specimens with 200-mm and 250-mm openings, respectively. A comparison of the results before and after the reinforcement modification indicates that, although the reinforcement rearrangement led to a more balanced stress distribution around the openings, the overall pattern of maximum principal stress distribution remained largely unchanged. The critical tensile stress regions continued to be concentrated primarily at the corners of the openings.

Examination of the quantitative results for the maximum principal stresses reveals that, following the reinforcement modification, these stresses increased by 4.9%, 7.5%, and 10.3% for specimens SO-200C-M, SO-200T-M, and SO-200B-M, respectively. Similarly, increases of 0.5% and 10.85% were observed for specimens SO-250T-M and SO-250B-M, whereas Specimen SO-250C-M exhibited a very slight reduction of 0.56% in the maximum principal stress. These findings indicate that, unlike the compressive stress field, reinforcement rearrangement does not directly reduce the peak tensile stress levels, and stress concentration at the opening corners remains a dominant characteristic of the structural behavior in this region.

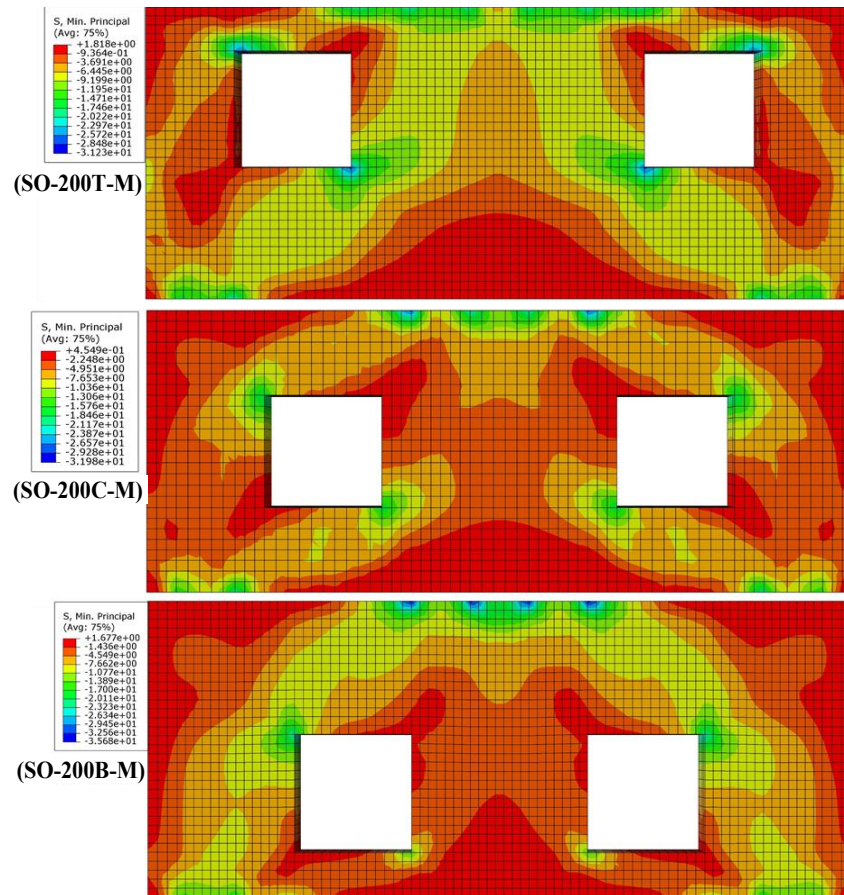


Fig. 15. Comparison of minimum principal stress fields in reinforced deep beam specimens with modified reinforcement layouts and 200×200 mm web openings.

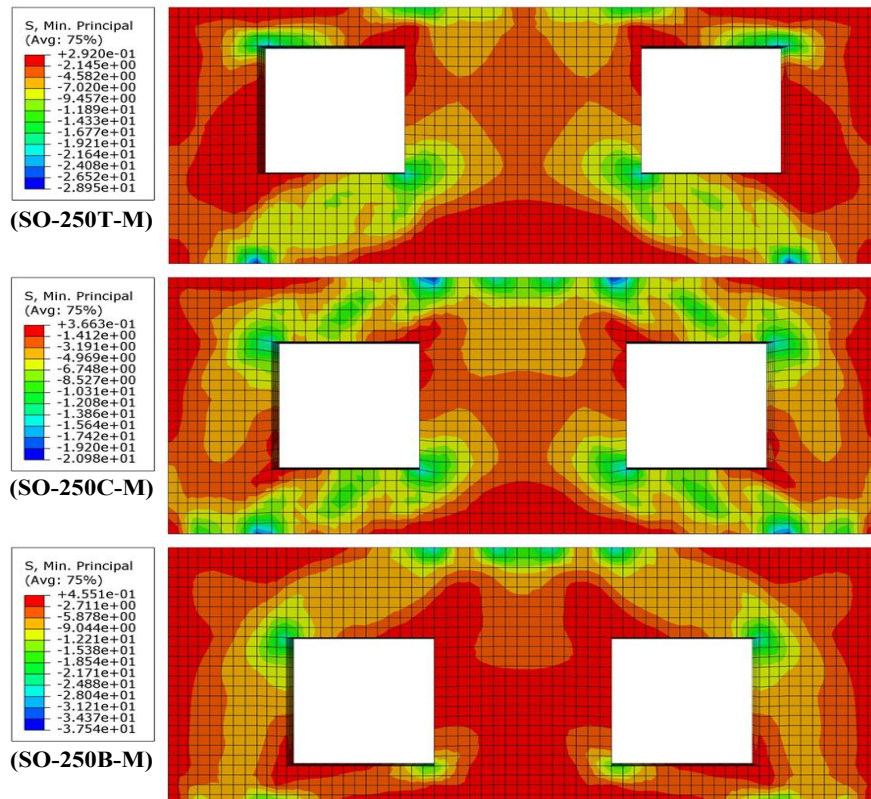


Fig. 16. Comparison of minimum principal stress fields in reinforced deep beam specimens with modified reinforcement layouts and 250×250 mm web openings.

This behavior can be attributed to the inherent nature of tensile stress concentrations within discontinuity regions. Even with modifications to the reinforcement layout and improvements in concrete confinement, the locations of critical tensile stresses remain largely unchanged. Nevertheless, a comparison of the contours presented in Figs. 17 and 18 demonstrates that the reinforcement rearrangement resulted in a more uniform stress distribution around the openings and facilitated the transfer of a portion of the localized stresses to adjacent regions. This stress redistribution, together with the observed increases in cracking load and ultimate

load-carrying capacity, indicates that although the effect of reinforcement modification on the peak tensile stress values is limited, it plays a significant role in improving the overall performance of the discontinuity region and enhancing the structural behavior of reinforced concrete deep beams with square openings.

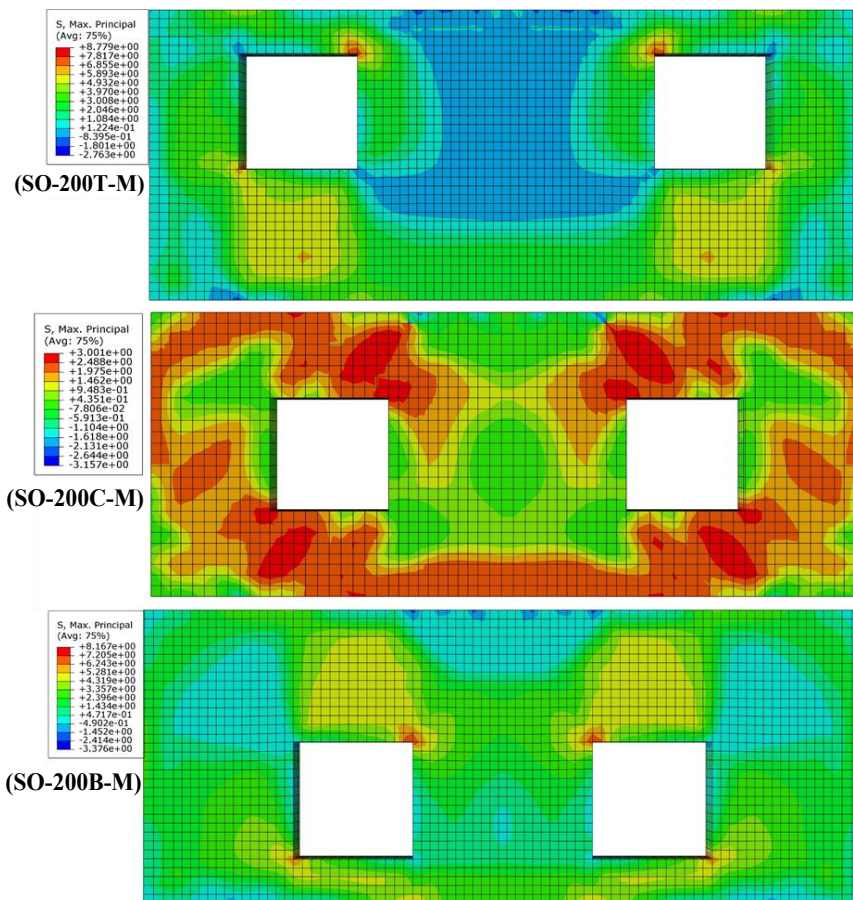


Fig. 17. Comparison of maximum principal stress fields in reinforced deep beam specimens with modified reinforcement layouts and 200 × 200 mm web openings.

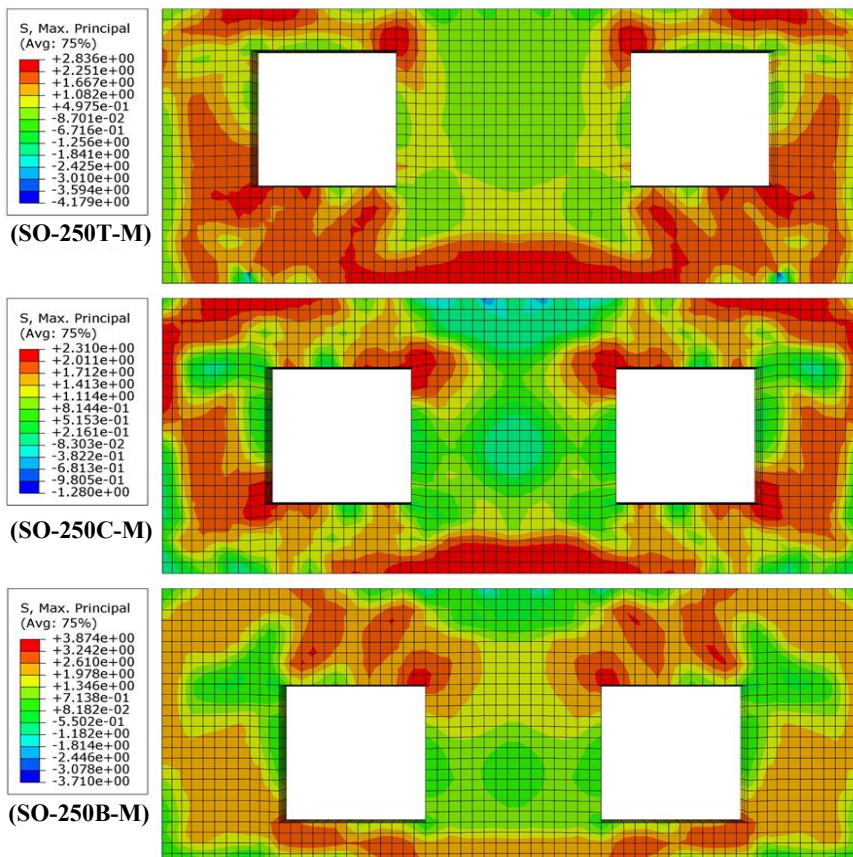


Fig. 18. Comparison of maximum principal stress fields in reinforced deep beam specimens with modified reinforcement layouts and 250 × 250 mm web openings.

For ease of comparison, the principal quantitative results obtained from the parametric study are summarized in Table 3. The table highlights the influence of opening size, opening location, and reinforcement rearrangement on the ultimate load capacity, cracking load, stress redistribution, and structural performance of the investigated deep beams.

Table 3. Summary of the influence of opening configuration and reinforcement detailing on the structural performance of reinforced concrete deep beams.

Specimen	Ultimate load (kN)	Cracking load (kN)	P_{cr} / P_u	Principal stress redistribution	Overall structural performance
SO200-C	159.64	47	0.29	Severe disruption of the principal compression strut	Most adversely affected by opening
SO200-T	218.08	82.28	0.37	Compression stresses largely bypass the opening	Least affected by opening
SO200-B	214.53	84.13	0.39	Partial disruption of the compression strut	Moderately affected by opening
SO250-C	104.53	39.26	0.37	Highly disturbed compression stress field	Most severely affected by opening size
SO250-T	127.54	42.86	0.34	Moderate stress redistribution	Moderately affected by opening size
SO250-B	136.76	53.98	0.39	Significant stress redistribution	Significantly affected by opening size
SO200-C-M	174.21	62.31	0.36	Improved stress redistribution	Structural performance substantially recovered
SO200-T-M	224.83	78.99	0.35	Slightly improved stress distribution	Structural performance slightly improved
SO200-B-M	221.39	91.95	0.41	Improved stress redistribution	Structural performance moderately improved
SO250-C-M	114.44	52.03	0.45	Significantly improved stress redistribution	Structural performance markedly improved
SO250-T-M	134.51	48.92	0.36	Moderately improved stress redistribution	Structural performance moderately improved
SO250-B-M	150.37	47.60	0.31	Enhanced stress redistribution around the opening	Structural performance significantly improved

4. Engineering implications

The findings of this study demonstrate that both the size and vertical location of web openings have a significant influence on the structural behavior of reinforced concrete deep beams. Within the investigated configurations, openings located at the mid-depth of the web caused the greatest disruption to the principal load-transfer path, resulting in reduced load-carrying capacity, increased stress concentrations, and more severe crack propagation. Furthermore, increasing the opening size intensifies these adverse effects and increases the sensitivity of the member to geometric discontinuities.

Conversely, the strategic redistribution of web reinforcement around the opening can significantly improve the structural performance of deep beams without requiring a substantial increase in the amount of reinforcing steel. The observed increases in cracking load, load-carrying capacity, deformation capacity, and the more uniform distribution of compressive stresses indicate that reinforcement detailing within discontinuity regions (D-regions) plays a critical role in controlling the behavior of deep beams containing web openings. Therefore, the results suggest that, in addition to the size and location of the opening, reinforcement detailing around the opening should be considered as an important design parameter for improving force transfer and structural performance. The proposed reinforcement arrangement provides a practical and economical approach for mitigating the adverse effects of square web openings in reinforced concrete deep beams without requiring significant modifications to member geometry. It should be noted that the proposed reinforcement detailing is intended as a practical detailing concept for deep beams with structural characteristics similar to those investigated in the present study. Further studies covering a wider range of beam geometries and loading conditions are recommended before generalizing the proposed detailing to broader design applications.

5. Conclusions

In this study, the structural behavior of reinforced concrete deep beams containing square web openings was investigated through nonlinear finite element analysis using the concrete damaged plasticity (CDP) model in ABAQUS [25]. The validated finite element model accurately reproduced the experimental load-displacement response, crack patterns, and ultimate load capacity, providing a reliable basis for the subsequent parametric investigation of deep beams with square web openings. The effects of opening size, opening location, and reinforcement rearrangement around the openings on crack propagation, stress distribution, and load-carrying capacity were examined. Based on the obtained results, the following conclusions can be drawn:

- The location of the opening was found to be one of the most influential parameters affecting the structural performance of deep beams. Openings located at the center of the shear span caused the greatest disruption to the main compression strut, resulting in the most severe disturbance of the internal force-transfer mechanism and the largest reduction in load-carrying capacity among the investigated specimens.
- Crack initiation and propagation were strongly dependent on both the size and vertical position of the opening. Due to the geometric discontinuity introduced by square openings, stress concentration occurred primarily at the opening corners, where diagonal shear cracks were initiated and subsequently propagated toward the loading and support regions.
- Increasing the opening size from 200 mm to 250 mm intensified stress concentration, increased the extent of diagonal cracking, and aggravated the disturbance of the compression stress field.

- Analysis of the principal stress fields demonstrated that the presence of square openings significantly altered the natural stress flow within the deep beam. Openings positioned along the main load-transfer path forced the compressive stresses to bypass the opening through narrow concrete zones, leading to localized stress concentrations and reduced load-carrying capacity.
- The proposed reinforcement rearrangement strategy, consisting of relocating web reinforcement around the opening and providing additional confinement reinforcement adjacent to critical regions, proved effective in enhancing the structural performance of the beams while requiring only a modest increase in steel consumption.
- The modified reinforcement layouts increased the ultimate load capacity by approximately 3-10% and delayed the initiation of diagonal cracking by 4.5-44%, depending on the opening size and location. Furthermore, the ratio of cracking load to ultimate load (P_{cr} / P_u) increased in all investigated specimens, indicating a delay in crack initiation and the onset of nonlinear behavior.
- Reinforcement modification improved the distribution of compressive stresses around the openings and reduced stress concentration in critical regions. Although its influence on the magnitude of peak tensile stresses was relatively limited, it significantly enhanced crack control, deformation capacity, and overall structural performance
- The effectiveness of the proposed reinforcement strategy became more pronounced as the opening size increased. This finding highlights the critical role of reinforcement detailing in discontinuity regions (D-regions), particularly in deep beams with large web openings where the disturbance of force flow is more severe.

Statements & declarations

Author contributions

Amir Keshavarz: Investigation, Formal analysis, Writing - Original Draft.

Ebrahim Fadaei: Methodology, Writing - Review & Editing.

Mohammad Ali Dastan Diznab: Project administration, Resources, Methodology, Conceptualization.

Seyed Hamid Hashemi: Methodology, Conceptualization.

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Data availability

The datasets analyzed in the current study are available from the corresponding author on reasonable request.

Declarations

The authors declare no conflict of interest.

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