

Size-Dependent Constitutive Model for Normal- and High-Strength Unconfined Concrete

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ABSTRACT

This study presents a unified stress-strain model for unconfined concrete under axial compression that accounts for key size-dependent effects and varying material strength levels. The proposed formulation is designed to overcome limitations in existing constitutive models, particularly their restricted applicability across different specimen geometries, strength classes, and their limited ability to consistently represent both pre-peak and post-peak behavior within a single coherent framework. The model adopts a two-stage representation of the compressive response, allowing independent yet physically consistent characterization of the nonlinear ascending branch and the strain-softening post-peak regime. The formulation explicitly incorporates the influence of concrete strength and specimen geometry, enabling the model to capture transitions from more ductile behavior in lower-strength concrete to increasingly brittle responses in higher-strength materials. The proposed approach is calibrated and validated using an extensive experimental database covering a wide range of concrete strengths, specimen dimensions, and aspect ratios compiled from multiple independent research programs. Comparisons with experimental stress-strain curves demonstrate that the model reliably reproduces both the peak response and the subsequent softening behavior across diverse testing conditions, including cases with pronounced geometric and material variability. The results confirm that the proposed formulation provides a simplified but robust representation of the compressive behavior of unconfined concrete, making it suitable for nonlinear structural analysis and performance-based engineering applications without the need for case-specific recalibration.

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1. Introduction

Concrete remains the most widely used construction material in civil infrastructure due to its high compressive strength, durability, versatility, and economic efficiency. The accurate characterization of its mechanical response under compression is fundamental for the analysis, design, assessment, and numerical simulation of reinforced concrete structures. In particular, the axial stress-strain relationship of unconfined concrete constitutes a key constitutive law employed in nonlinear finite element analyses, performance-based structural design, and the evaluation of structural safety under extreme loading conditions. Since the pioneering work of Hognestad [1], numerous analytical models have been proposed to represent the compressive behavior of concrete, including those developed by Kent and Park [2], Popovics [3], Mander et al. [4], Collins et al. [5], Légeron and Paultre [6], Karthik Madhu and Mander John [7], Lim and Ozbakkaloglu [8], Yang et al. [9], Shayanfar et al. [10]. Although these models have significantly advanced the understanding of concrete constitutive behavior, the increasing availability of experimental evidence

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covering broader strength ranges and specimen dimensions has highlighted the need for more versatile and physically representative formulations. In particular, the influence of size effect on compressive behavior, together with the transition from normal- to high-strength concrete, requires constitutive models capable of providing reliable predictions across a wide spectrum of material and geometric conditions.

The development of stress-strain models for unconfined concrete has evolved substantially over the past several decades; however, several limitations remain that restrict the accuracy, robustness, and practical applicability of existing formulations. A primary concern is the mathematical complexity adopted by some constitutive models. To improve curve-fitting performance, several researchers have introduced formulations involving numerous empirical coefficients, higher-order functions, or iterative calibration procedures (e.g., Karthik Madhu and Mander John [7]). Although these approaches may provide satisfactory agreement with specific experimental datasets, they often reduce transparency regarding the physical meaning of model parameters and complicate their implementation in design-oriented calculations and nonlinear numerical analyses. From an engineering perspective, a constitutive model should not only reproduce experimental observations with sufficient accuracy but should also maintain computational efficiency, numerical stability, and ease of calibration using readily available material properties. Another important limitation relates to the use of a single analytical function to represent the complete compressive response of concrete, including both the pre-peak hardening and post-peak softening regions. The mechanical mechanisms governing these two stages are fundamentally different. The ascending branch is primarily controlled by microcrack initiation, progressive crack accumulation, and material compaction, whereas the descending branch is dominated by crack localization, fracture propagation, aggregate interlock degradation, and progressive loss of load-carrying capacity (Tariq et al. [11], Shayanfar et al. [12]). Consequently, enforcing a single continuous mathematical expression over the entire stress-strain domain requires the simultaneous satisfaction of multiple boundary conditions associated with both regions. Such constraints often reduce the flexibility of the model to independently capture the curvature of the ascending branch and the rate of post-peak deterioration. This limitation becomes particularly evident in the softening regime, where small variations in material properties or specimen geometry can produce significant changes in the shape of the stress-strain response (Shayanfar et al. [13, 14]). As a result, models employing a unified function frequently exhibit reduced accuracy in predicting strain-softening behavior, which is critical for evaluating ductility, energy dissipation capacity, and failure progression in concrete structures. The applicability of many existing models is further constrained by their calibration range. A considerable proportion of constitutive formulations were developed using experimental datasets dominated by normal-strength concrete, typically within a relatively narrow strength interval. However, the mechanical response of concrete changes significantly with increasing compressive strength (e.g., Yang et al. [9]). High-strength concrete generally exhibits a denser microstructure, reduced internal porosity, and improved matrix-aggregate bonding, resulting in higher peak stresses but also increased brittleness and steeper post-peak degradation (Eid et al. [15], Shayanfar et al. [16–19]). Consequently, parameters calibrated for normal-strength concrete cannot necessarily reproduce the behavior of high-strength concrete with sufficient accuracy. Similarly, constitutive parameters derived from high-strength concrete datasets may overestimate brittleness when applied to lower-strength materials. This dependence on strength class limits the generality of many existing formulations and necessitates the development of unified models capable of accurately representing a broad spectrum of concrete strengths without requiring separate calibration procedures. A further limitation concerns the insufficient consideration of size effect in many stress-strain relationships. Concrete is a quasi-brittle material whose mechanical response is strongly influenced by specimen dimensions due to the heterogeneous nature of its internal structure and the localization of fracture processes. Experimental investigations (e.g., Gholampour et al. [20]) have consistently demonstrated that increasing specimen size leads to reductions in nominal compressive strength and alterations in deformation capacity. These effects originate from the statistical distribution of flaws, the development of localized fracture zones, and the non-uniform dissipation of fracture energy within the material (Shayanfar and Barros Joaquim [21]). Larger specimens tend to exhibit more pronounced strain localization and a more brittle post-peak response than smaller specimens (Shayanfar et al. [22], Watanabe et al. [23]). Nevertheless, a substantial number of constitutive models have been developed primarily from tests performed on standard cylinders or relatively small laboratory specimens, implicitly assuming geometric independence of the stress-strain response. Such an assumption can result in significant prediction errors when the models are applied to structural members with dimensions substantially different from those used during calibration (Shayanfar et al. [24, 25]). Although some researchers have attempted to incorporate size-effect phenomena into constitutive formulations, these approaches frequently involve complex analytical procedures, multiple correction factors, or intermediate calculations that reduce their practicality for engineering applications. In many cases, the implementation of size-dependent formulations requires the determination of parameters that are not routinely measured in experimental programs or available in design specifications (Paultre et al. [26], Sadeghpour Haji et al. [27]). Consequently, despite their improved theoretical rigor, such models may be difficult to integrate into structural analysis software or everyday engineering practice. The challenge therefore lies in developing a formulation that captures the essential physics of size-dependent behavior while maintaining a level of simplicity suitable for widespread application. Finally, the predictive reliability of many existing models is influenced by the scope and quality of the experimental databases used during their development. Several constitutive relationships were calibrated using relatively limited datasets covering a restricted range of concrete strengths, specimen dimensions, and geometric configurations. Such datasets may not adequately capture the inherent variability associated with concrete behavior, including differences in aggregate characteristics, curing conditions, specimen geometry, loading procedures, and material composition. As a consequence, the resulting formulations may exhibit satisfactory performance within the calibration domain but reduced predictive capability when extrapolated to conditions outside that range. The increasing availability of extensive experimental databases now provides an opportunity to establish statistically robust constitutive relationships whose parameters reflect a broader spectrum of material and geometric characteristics. Collectively, these limitations highlight the need for a new generation of stress-strain models that integrate physical realism, size-effect sensitivity, computational simplicity, and broad applicability within a unified constitutive framework.

To address these challenges, the present study proposes a new size-dependent stress-strain model for unconfined concrete subjected to axial compression. The proposed methodology adopts a two-branch constitutive representation consisting of a polynomial-based ascending branch and a logistic-type descending branch, enabling independent calibration of the pre-peak and post-peak responses and thus improving the accuracy of softening behavior prediction. The model explicitly incorporates the effects of concrete strength, specimen dimensions, and aspect ratio through data-driven formulations calibrated using an extensive experimental database covering a wide range of concrete classes and geometrical configurations. In addition, size-effect formulations are incorporated directly into the determination of peak stress and corresponding strain, ensuring consistency with experimentally observed scaling phenomena. The resulting model aims to provide a unified, computationally efficient, and statistically robust constitutive relationship capable of accurately reproducing the stress-strain response of unconfined concrete, from initial loading to post-peak degradation, while maintaining applicability across different specimen sizes and strength levels.

2. Estimation of peak stage parameters

To assess the compressive behavior of unconfined concrete, it is first necessary to define its peak stage properties, specifically the compressive strength (f_{c0}) and the corresponding axial strain (ε_{c0}), which serve as fundamental input parameters.

Experimental investigations (e.g., Watanabe et al. [23]) have shown that the mechanical properties of large-scale concrete columns, namely f_{c0} and ε_{c0} , are significantly lower than those obtained from standard cylindrical specimens (diameter $D = 150$ mm and height $L = 300$ mm). This discrepancy is mainly attributed to the size effect phenomenon. In the present work, the formulation proposed by Yang et al. [9] is adopted to estimate f_{c0} based on the compressive strength of standard cylinders (f'_{c0}), expressed as follows (with minor rearrangement):

$$f_{c0} = \left[0.63 + 0.9 \sqrt{\frac{\left(\frac{L}{D}\right)^{-0.6}}{1+0.017D}} \right] f'_{c0} \approx 1.063 \left(\frac{150}{D}\right)^{0.122} \left(\frac{D}{L}\right)^{0.088} f'_{c0} \quad (1)$$

In addition, previous experimental evidence (e.g., Dahl [28]) indicates a clear relationship between f_{c0} and the corresponding ε_{c0} with ε_{c0} generally increasing as f_{c0} increases. Beyond this dependency, Jansen Daniel and Shah Surendra [29] reported that the column aspect ratio (L/D) also plays a significant role in influencing ε_{c0} . In this study, to estimate ε_{c0} while accounting for size effects, a comprehensive database comprising 604 unconfined concrete specimens was assembled, as summarized in Table 1. Based on regression analysis of this dataset, the following empirical expression is proposed as a function of f_{c0} and the aspect ratio L/D :

$$\varepsilon_{c0} = 0.0011 \left(\frac{f_{c0} D}{L} \right)^{0.25} \quad (2)$$

The predictive capability of Eq. 2 against the collected experimental data ($\varepsilon_{c0}^{Ana}/\varepsilon_{c0}^{Exp}$) illustrated in Table 2, considering the variables f_{c0} , L/D , and D . The statistical evaluation presented in Table 2 confirms that the proposed model provides satisfactory accuracy. Moreover, its performance is superior to existing formulations proposed by Lim Jian and Ozbakkaloglu [30] and Popovics [3]. Notably, the influence of D in Eq. 2 is relatively insignificant; hence, it is neglected in the present formulation.

Table 1. Summary of the collected database.

Number of datasets		f_{c0} range (MPa)	ε_{c0} range	L/D range	D range (mm)
604	Min.	11.6	0.0012	1.0	54
	Max.	204.0	0.0051	5.5	500
	Mean	51.2	0.0025	2.2	133.4
	CoV	0.534	0.233	0.297	0.339

Table 2. Comparative performance of Eq. 2.

Model	Expression	Test Data	Assessment indicators		
			Mean	SD	MAPE
Proposed model	$\varepsilon_{c0} = 0.0011 \left(\frac{f_{c0} D}{L} \right)^{0.25}$	604	0.987	0.180	0.138
Lim Jian and Ozbakkaloglu [30]	$\varepsilon_{c0} = 0.001 f_{c0}^{0.225} \left(\frac{152}{D} \right)^{0.1} \left(\frac{2D}{L} \right)^{0.13}$	604	0.988	0.186	0.143
Popovics [3]	$\varepsilon_{c0} = 0.000937 f_{c0}^{0.25}$	604	1.009	0.192	0.147

Note: Mean = mean prediction-to-experiment ratio; SD = standard deviation of the prediction-to-experiment ratio; MAPE = mean absolute percentage error.

3. Proposed stress-strain relationship

The mechanical response of unconfined concrete under axial compression is typically characterized by a nonlinear stress-strain relationship governed by progressive microcracking, stiffness degradation, and eventual peak stress attainment. Capturing this behavior with sufficient accuracy while maintaining analytical simplicity is essential for practical constitutive modeling and subsequent structural analysis. In this study, based on observed experimental trends, a new stress-strain formulation is proposed in which the response is decomposed into two distinct stages: a nonlinear ascending branch up to the peak point and a simplified linear

descending branch thereafter.

3.1. Pre-peak stress-strain response

For the ascending branch of the stress-strain curve, a simplified polynomial function is adopted in the form:

$$\frac{f_c}{f_{c0}} = A_1 \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right)^{n_{d1}} + A_2 \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right) + A_3 \text{ for } \varepsilon_c \leq \varepsilon_{c0} \quad (3)$$

where n_{d1} defines the degree of the polynomial (non-negative integer). A_1 to A_3 are the coefficients of the polynomial. To evaluate these coefficients, the following three boundary conditions are imposed:

$$\text{I) } \frac{\varepsilon_c}{\varepsilon_{c0}} = 0 \rightarrow \frac{f_c}{f_{c0}} = 0 \quad (4)$$

$$\text{II) } \frac{\varepsilon_c}{\varepsilon_{c0}} = 1 \rightarrow \frac{f_c}{f_{c0}} = 1 \quad (5)$$

$$\text{III) } \frac{\varepsilon_c}{\varepsilon_{c0}} = 1 \rightarrow \frac{d\left(\frac{f_c}{f_{c0}}\right)}{d\left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)} = 0 \quad (6)$$

These boundary conditions are established based on the fundamental mechanical behavior of unconfined concrete under compressive loading. Eq. 6 enforces a zero-slope condition at the peak point, confirming that the derivative of the relative stress ratio with respect to the relative strain ratio equals zero at the apex. This guarantees a smooth mathematical transition from the pre-peak ascending parabola to the subsequent post-peak softening region. The coefficients of A_1 to A_3 can be, accordingly, derived as $A_1 = \frac{1}{(1-n_{d1})}$, $A_2 = -\frac{n_{d1}}{(1-n_{d1})}$ and $A_3 = 0$. Thus, Eq. 3 can be rewritten based on the boundary conditions:

$$f_c = \left[\frac{1}{1-n_{d1}} \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right)^{n_{d1}} - \frac{n_{d1}}{1-n_{d1}} \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right) \right] f_{c0} \text{ for } \varepsilon_c \leq \varepsilon_{c0} \quad (7)$$

Fig. 1a compares f_c/f_{c0} versus $\varepsilon_c/\varepsilon_{c0}$ relations reported from a set of experimental unconfined concrete specimens with a concrete strength class of 34 MPa with those obtained analytically from Eq. 7. As demonstrated in Fig. 1a, the pre-peak response of normal-strength unconfined concrete specimens, spanning concrete strength classes of 32 to 36 MPa and core dimensions (D) from 76 to 200 mm, is accurately captured by setting the polynomial degree parameter n_{d1} to 2.55. The analytical model closely matches the initial tangent stiffness and tracks the progressive, microcrack-induced nonlinearity up to the peak stress threshold (f_{c0}). This strong correlation across diverse experimental programs confirms that Eq. 7 effectively neutralizes specimen size variations when calibrated with the appropriate shape factor. Conversely, Fig. 1b evaluates the model's performance against high-strength concrete specimens ranging from 58 to 62 MPa. To accommodate the inherent mechanical differences of higher strength classes, the shape parameter n_{d1} must be increased to 4.25 based on the experimental data. From a material mechanics perspective, this elevated n_{d1} value represents a more linear stress-strain trajectory during the early-to-mid loading stages, followed by a sharper curvature change just prior to reaching peak capacity. This mathematical shift physically corresponds to the delayed microcracking and more brittle pre-peak behavior characteristic of high-strength concrete matrices, which exhibit noticeably less relative stress (f_c/f_{c0}) at given intermediate strain ratios ($\varepsilon_c/\varepsilon_{c0}$) compared to lower strength counterparts. The excellent agreement demonstrated in both figures confirms that by treating n_{d1} as a variable parameter rather than a fixed constant (such as the traditional n_{d1} adopted in classic parabolic models), the formulation preserves high predictive accuracy across distinct concrete strength spectra without losing its simplified mathematical format.

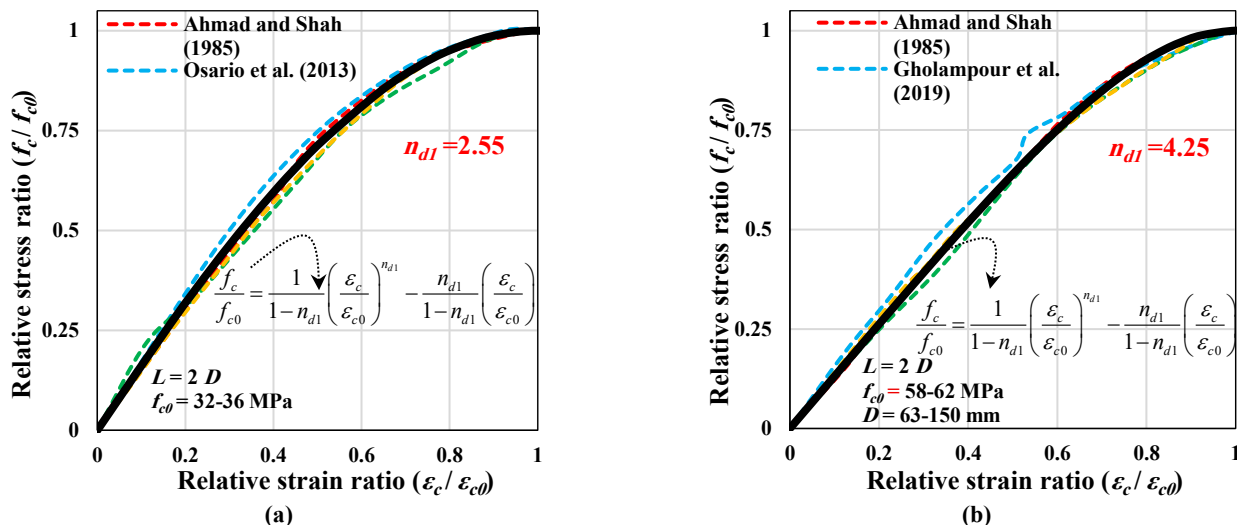


Fig. 1. Variation of f_c^{Exp}/f_{c0} with respect to $\varepsilon_c/\varepsilon_{c0}$: (a) Normal-strength concrete; and (b) High-strength concrete.

Note: Test data obtained from Ahmad and Shah [31], Dahl [28], Eid et al. [15], Osorio et al. [32], Paultre et al. [26], and Gholampour et al. [20].

In order to determine n_{d1} as an input parameter, a large test database of unconfined concrete columns was provided. For this purpose, n_{d1}^{Exp} was derived through the implementation of Eq. 7 on the experimental stress-strain relation so that Eq. 7 could closely predict the entire pre-peak stress-strain relation of counterparts. Fig. 2 presents the relations of n_{d1}^{Exp} with respect to the key variables of f_{c0} , L/D , and D . It can be seen in Fig. 2a that f_{c0} is the most significant variable in n_{d1} so that by increasing f_{c0} , n_{d1}^{Exp} increases noticeably. Fig. 2b reveals that L/D has a slight effect on n_{d1}^{Exp} . Furthermore, by increasing the normalized column dimension ($D/150$), n_{d1}^{Exp} decreases significantly (Fig. 2c).

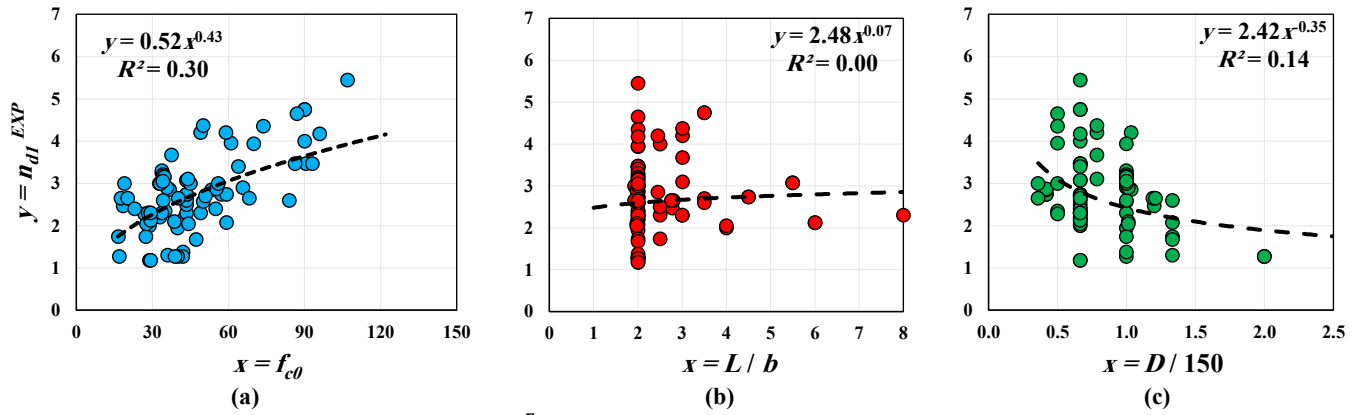


Fig. 2. Variation of n_{d1}^{Exp} with respect to: (a) f_{c0} ; (b) L/D ; and (c) $D/150$.

It is worth noting that the influence of L/D on n_{d1} is relatively weak compared with that of concrete strength. This behavior is consistent with the mechanics of the pre-peak response, during which deformation remains nearly uniform, and damage is still distributed throughout the specimen. Based on statistical analysis of the influence of key variables on n_{d1}^{Exp} , a new regression-based developed formulation is suggested as:

$$n_{d1} = \frac{0.77f_{c0}^{0.4}\left(\frac{L}{D}\right)^{0.1}}{1 + \frac{D}{300}} \quad (8)$$

Fig. 3 explicitly quantifies the predictive performance of this developed formulation by plotting the experimentally derived shape factor n_{d1}^{Exp} against the analytically calculated counterpart n_{d1}^{Ana} . The predictive reliability is statistically substantiated across 85 test data points, demonstrating a highly favorable alignment along the 1:1 ideal prediction line. The robustness of the proposed empirical equation is further confirmed by its statistical indicators, which reveal a mean ratio (Mean) of 1.047, a coefficient of variation (CoV) of 0.272, and a mean absolute percentage error (MAPE) of 0.226. The excellent agreement demonstrated in both figures confirms that by treating n_{d1} as a variable parameter determined via Eq. 8, the formulation preserves high predictive accuracy across distinct concrete strength spectra and specimen geometries without losing its simplified mathematical format.

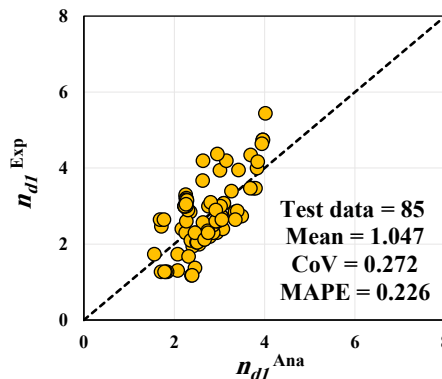


Fig. 3. Examination of n_{d1}^{Exp} versus n_{d1}^{Ana} relation.

It is noteworthy that in the case of adopting a second-degree polynomial function ($n_{d1} = 2$) to simulate the pre-peak response of unconfined concrete, by using the following three boundary conditions presented in Eqs. 4 to 6, the stress-strain relation could be determined (as proposed by Hognestad [1], Kent and Park [2], Vecchio and Collins [33], and Légeron and Paultre [6]):

$$f_c = \left[2 \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right) - \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] f_{c0} \text{ for } \varepsilon_c \leq \varepsilon_{c0} \quad (9)$$

Accordingly, owing to its mathematical form, this formulation assumes a fixed normalized curvature of the ascending branch ($n_{d1} = 2$), irrespective of concrete strength or specimen geometry. Although the peak strain ε_{c0} may vary with f_{c0} , L/D , and D , modifying the peak strain only rescales the strain axis and does not alter the intrinsic evolution of the normalized stress-strain relationship. Experimental results (Figs. 2 and 3) demonstrate that these variables influence not only the peak coordinates but also the rate at which stress develops before the peak, reflecting changes in stiffness degradation associated with different material strengths and specimen geometries. Consequently, expressing their influence solely through ε_{c0} is insufficient to reproduce the experimentally observed changes in the normalized pre-peak response. Introducing n_{d1} as a variable parameter provides an

independent degree of freedom to describe these changes in the normalized constitutive response while preserving the simplicity of the polynomial formulation. Consequently, adopting a constant value of $n_{d1} = 2$ may lead to misleading predictions, whereas the proposed formulation preserves both simplicity and predictive accuracy by expressing n_{d1} as a function of the governing variables through Eq. 8.

3.2. Post-peak stress-strain response

The post-peak response of unconfined concrete under axial compression is governed by a strongly nonlinear softening behavior associated with progressive damage evolution. After reaching the peak stress, the material response is dominated by the development and coalescence of distributed microcracks into localized macro-cracks, ultimately forming a distinct fracture zone. This localization mechanism leads to a pronounced reduction in load-carrying capacity, accompanied by rapid stiffness degradation and strain concentration within the damaged region. From a continuum mechanics standpoint, deformation beyond the peak stage is no longer homogeneously distributed across the specimen. Instead, inelastic deformation becomes increasingly confined to the fracture zone, while the surrounding material primarily undergoes elastic unloading. To capture this transition in a consistent and normalized framework, a non-dimensional relative strain parameter, denoted as X_ε , is introduced. This parameter transforms the post-peak degradation process into a unified dimensionless space, facilitating generalized modeling across different concrete strengths and geometries. This ratio is strategically defined such that $X_\varepsilon = 1$ when the propagating compressive strain reaches a critical milestone of ε_{ci0} , representing the specific post-peak axial strain at which the concrete's remaining capacity drops exactly to half of its peak strength ($0.5f_{c0}$). Utilizing this boundary condition, the multi-variable relationship between the relative stress ratio (f_c/f_{c0}) and the normalized strain ratio (X_ε) is mathematically proposed via the following logistic-based type function:

$$\frac{f_c}{f_{c0}} = \frac{1}{X_\varepsilon^{n_{d2}} + 1} \quad \text{for } \varepsilon_c \geq \varepsilon_{c0} \quad (10)$$

in which:

$$X_\varepsilon = \frac{\varepsilon_c - \varepsilon_{c0}}{\varepsilon_{ci0} - \varepsilon_{c0}} \quad (11)$$

From a material mechanics and continuum perspective, the milestone $X_\varepsilon = 1$ (where $\varepsilon_c = \varepsilon_{ci0}$) physically isolates the mathematical inflection point of this logistic sigmoidal function. At this stage, the response reaches $0.5f_{c0}$, which physically represents the point of maximum degradation rate, where the slope $df_c/d\varepsilon$ attains its most negative value. This inflection behavior captures the transition from distributed damage to highly localized fracture-controlled softening. To accommodate distinct material characteristics, the exponent n_{d2} is introduced as an empirical shape-adjustment parameter. This parameter governs the curvature and steepness of the post-peak branch, enabling the model to adapt to different concrete strengths and specimen aspect ratios. In this way, n_{d2} controls the rate of strain softening and indirectly reflects changes in fracture energy dissipation related to varying material configurations.

Fig. 4a presents the variation of the normalized stress ratio f_{c0} with respect to X_ε , obtained from unconfined concrete specimens with an average concrete strength of 62 MPa and a constant slenderness ratio of $L/D = 2$, but with different cross-sectional dimensions D . As shown, the proposed model in Eq. 10 reproduces the experimental softening response with good agreement when the shape parameter is calibrated as $n_{d2} = 1.88$. In this case, the model captures both the initial steep post-peak drop and the subsequent gradual decay in residual capacity, indicating that the logistic formulation is capable of representing the transition from distributed damage to localized fracture. Fig. 4b shows the influence of concrete strength on the post-peak curvature. For the lower strength class of 34 MPa, the experimental response exhibits a less pronounced curvature and a more gradual reduction in stress. This reflects a more distributed cracking process and higher strain capacity prior to complete localization. Accordingly, a lower value of $n_{d2} = 1.62$ is required to reproduce this smoother softening response. The reduction in n_{d2} with decreasing strength is consistent with the increased ductility and delayed localization typically observed in lower-strength concretes. The effect of geometry is examined in Fig. 4c in terms of the column aspect ratio L/D . Increasing L/D leads to a more pronounced post-peak softening response, associated with a stronger tendency toward strain localization over a shorter effective crushing zone. This behavior is reflected in a systematic increase of the calibrated parameter n_{d2} , from 1.47 for $L/D = 2$ to 2.91 for $L/D = 4$. The increase in n_{d2} indicates a steeper decay in stress after peak, consistent with reduced confinement stability and more abrupt fracture development in slender specimens. Overall, Eq. 10 provides a consistent representation of the post-peak behavior of unconfined concrete across different strength levels and geometries. The model effectively links the curvature of the softening branch to both material properties and structural slenderness through the parameter n_{d2} . In this study, both ε_{ci0} and n_{d2} were identified through back-analysis by fitting Eq. 10 to the experimental stress-strain curves to obtain parameter sets that best reproduce the observed post-peak response.

To fully define the proposed logistic-based softening formulation, the two governing parameters controlling the location and shape of the post-peak branch, namely the normalized half-capacity strain ratio ($\varepsilon_{ci0}/\varepsilon_{c0}$) and the curvature parameter (n_{d2}), must be expressed as functions of measurable material and geometric properties. Accordingly, the experimentally calibrated values of ε_{ci0} and n_{d2} , obtained through the back-analysis procedure described previously, were systematically examined using the complete database of unconfined concrete specimens. Particular attention was given to the influence of f_{c0} , L/D , and D , which are known to influence fracture localization, size effect, and post-peak deformation capacity.

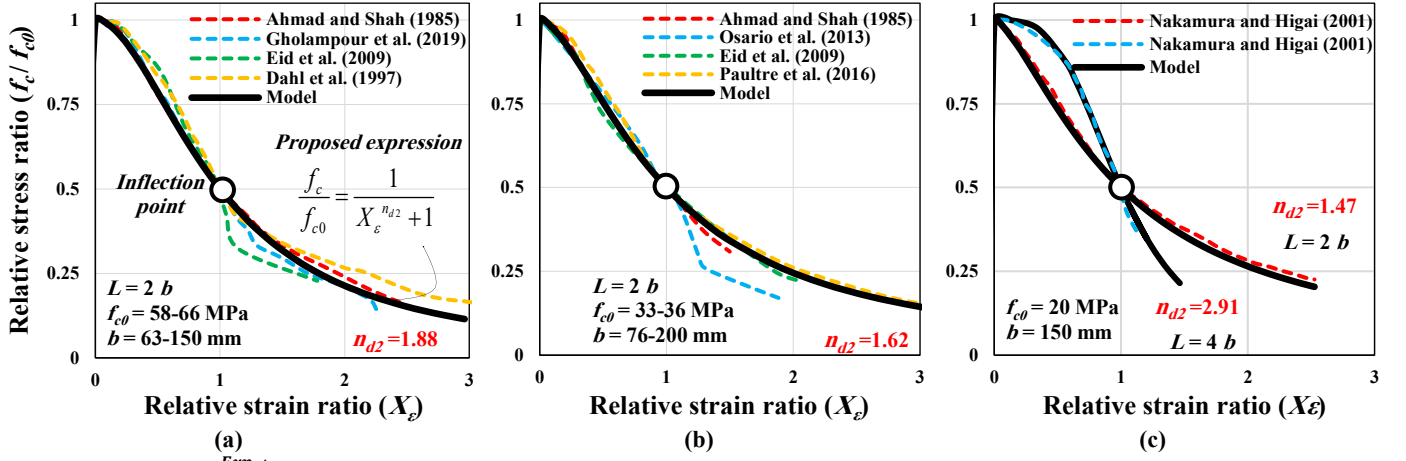


Fig. 4. Variation of f_c^{Exp}/f_{c0} with respect to X_{ϵ} : (a) effect of specimen size; (b) effect of concrete compressive strength; and (c) effect of specimen aspect ratio.

Note: Test data obtained from Ahmad and Shah [31], Dahl [28], Nakamura and Higai [34], Eid et al. [15], Osorio et al. [32], Paultre et al. [26], and Gholampour et al. [20].

Fig. 5 presents the variation of the experimentally extracted strain ratio ($\epsilon_{ci0}/\epsilon_{c0}$) with respect to the considered variables. As shown in Fig. 5a, a clear decreasing trend is observed with increasing concrete strength. Although the database exhibits the typical scatter associated with post-peak measurements, the overall power-law trend remains evident. This behavior is consistent with the fracture characteristics of higher-strength concrete, which generally possesses a denser microstructure and reduced crack tortuosity. Consequently, once peak stress is attained, crack propagation becomes more localized and unstable, causing the stress to decline to 50% of its peak value within a relatively smaller strain interval. As a result, the $\epsilon_{ci0}/\epsilon_{c0}$ ratio decreases with increasing strength. A similar trend can be observed in Fig. 5b with respect to the aspect ratio (L/D), where the relation indicates a pronounced reduction in $\epsilon_{ci0}/\epsilon_{c0}$ as the specimen becomes more slender. From a mechanics standpoint, post-peak deformation in unconfined concrete is governed primarily by strain localization within a relatively short fracture process zone. While this localized region undergoes significant inelastic deformation, the remaining portion of the specimen experiences progressive elastic unloading. Therefore, as L/D increases, the localized deformation becomes distributed over a larger gauge length, reducing the apparent average ductility measured at the specimen level and consequently lowering the normalized half-capacity strain ratio. The influence of specimen size is illustrated in Fig. 5c. When considered independently, the normalized dimension ($D/150$) exhibits a relatively weak positive correlation. However, the low coefficient of determination indicates that the apparent trend is strongly influenced by interactions with the other governing variables. In fact, when the complete database is analyzed simultaneously through multi-variable regression, the size effect contribution becomes negative. This outcome is physically reasonable, as larger specimens contain a greater volume of material defects and microstructural heterogeneities, promoting earlier crack coalescence and more rapid localization after peak stress. Consequently, the strain required to reach 50% of the peak stress tends to decrease with increasing specimen size.

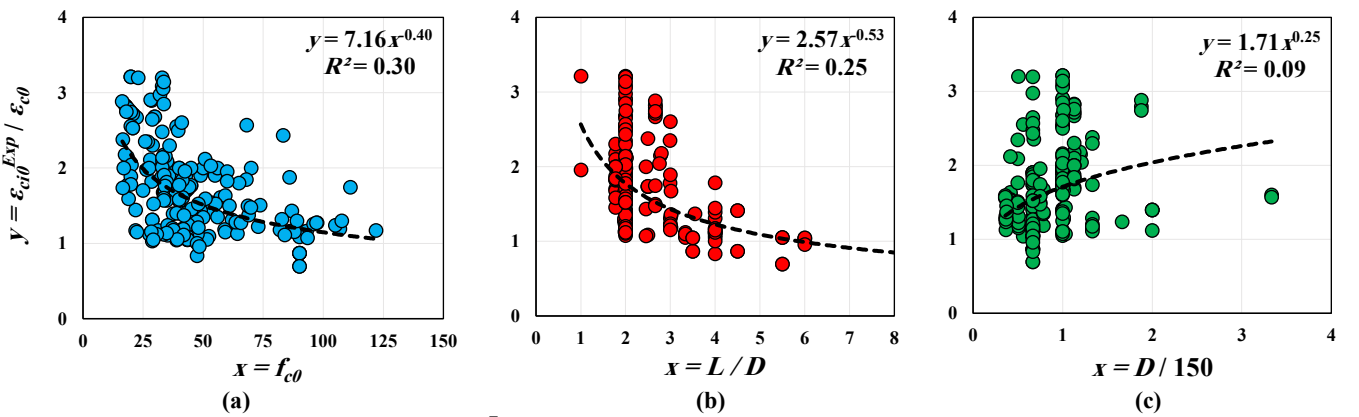


Fig. 5. Variation of $\epsilon_{ci0}^{Exp}/\epsilon_{c0}$ with respect to: (a) f_{c0} ; (b) L/D ; and (c) $D/150$.

multi-variable regression analysis was conducted by coupling the cumulative dependencies of all three variables simultaneously. By prioritizing the coupled power-law interactions, based on regression analysis, the predictive formulation for the $\epsilon_{ci0}^{Exp}/\epsilon_{c0}$ ratio can be obtained as:

$$\frac{\epsilon_{ci0}}{\epsilon_{c0}} = 23.4 f_{c0}^{-0.59} \left(\frac{L}{D}\right)^{-0.56} \left(\frac{D}{150}\right)^{-0.1} \quad (12)$$

To rigorously confirm the predictive accuracy of the proposed multi-variable empirical formulation for $\epsilon_{ci0}/\epsilon_{c0}$ ratio, the analytical results calculated via Eq. 12 are directly verified against 217 distinct experimental test data points in Fig. 6. The data points cluster tightly along the 1:1 ideal equality line, yielding an exceptional Mean value of 0.987, which demonstrates that the formulation converges to the mechanical center of the experimental cloud with negligible systematic bias. Furthermore, despite the

high inherent scatter and chaotic macro-cracking kinetics characteristic of unconfined concrete strain-softening, the model maintains a robust CoV of 0.210 and a MAPE of 0.160. This high degree of convergence scientifically validates the simultaneous power-law coupling of f_{c0} , L/D and $D/150$, proving the suitable performance of Eq. 12.

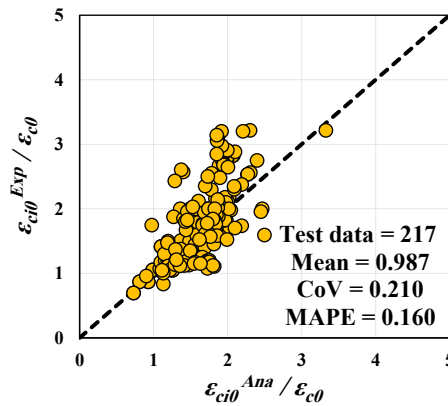


Fig. 6. Predictive performance of Eq. 12.

The experimentally calibrated values of the post-peak curvature parameter (n_{d2}^{Exp}) are examined in Fig. 7. Unlike $\varepsilon_{ci0}/\varepsilon_{c0}$, which primarily controls the position of the softening branch, n_{d2} governs its curvature and therefore directly influences the rate of stress degradation following peak loading. As shown in Fig. 7a, the influence of concrete strength is relatively modest. Nevertheless, the observed trend indicates that increasing strength modifies the curvature of the softening branch. Nevertheless, the observed trend suggests that increasing concrete strength systematically modifies the shape of the post-peak response. Since higher-strength concretes typically experience more rapid crack localization and reduced post-peak deformation capacity, a substantial portion of the strength loss occurs within a relatively narrow strain interval immediately after peak stress.

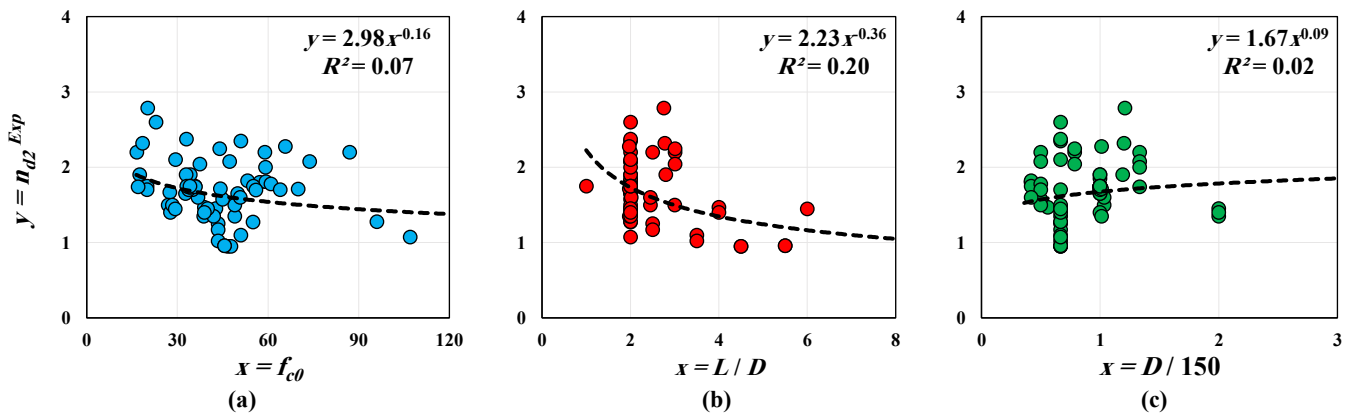


Fig. 7. Variation of n_{d2}^{Exp} with respect to: (a) f_{c0} ; (b) L/D ; and (c) $D/150$.

Consequently, the softening branch reaches its inflection region earlier and exhibits a less extended transition zone. Within the proposed logistic formulation, this behavior is captured through a reduction in n_{d2} , indicating that the curvature of the normalized softening response is sensitive to the degree of brittleness associated with increasing concrete strength. Fig. 7b demonstrates that the aspect ratio (L/D) is the most influential variable governing n_{d2} . From a mechanical standpoint, post-peak deformation in unconfined concrete is concentrated within a localized fracture zone whose characteristic length is substantially smaller than the overall specimen height. As the aspect ratio increases, the ratio between the fracture-zone length and the specimen length decreases, causing the inelastic deformation to become increasingly localized relative to the total specimen dimension. Consequently, a larger proportion of the specimen undergoes elastic unloading while only a limited region continues to accumulate damage and inelastic strains. This mechanism produces a more abrupt transition from distributed damage to localized crushing, resulting in a sharper change in the slope of the stress-strain curve immediately after the peak point. In the normalized space adopted by the proposed model, this behavior manifests as an increase in the curvature of the softening branch around the inflection region. Therefore, larger aspect ratios require systematic adjustment of n_{d2} to reproduce the experimentally observed localization-induced stress degradation. The strong statistical correlation observed in Fig. 7b confirms that the evolution of post-peak curvature is governed primarily by geometric effects associated with strain localization rather than by material strength alone. The influence of normalized specimen size is presented in Fig. 7c. Although a slight positive trend is observed, it indicates a negligible contribution. Therefore, inclusion of this parameter in the final model provides little improvement in predictive accuracy while increasing model complexity. Consequently, the size term was excluded from the final regression formulation.

Based on the preceding observations, a simplified multi-variable regression analysis was conducted using the two dominant variables, namely concrete compressive strength and specimen aspect ratio. The resulting expression is given by:

$$n_{d2} = 4.2f_{c0}^{-0.17} \left(\frac{L}{D}\right)^{-0.36} \quad (13)$$

To assess the predictive capability of the proposed expression for the post-peak curvature parameter n_{d2} , the analytical values calculated using Eq. 13 are compared against 71 experimentally calibrated values obtained through back-analysis, as shown in Fig. 8. Overall, the predictions exhibit a close agreement with the experimental counterparts, with the data points distributed reasonably around the 1:1 line and without any noticeable systematic bias over the considered range of parameters. The statistical indicators further confirm the reliability of the proposed formulation. A mean value of 1.030 indicates that Eq. 13 provides nearly unbiased predictions, with only a slight tendency toward conservative estimation. In addition, the relatively low CoV = 0.195 and MAPE = 0.165 demonstrate that the scatter between predicted and experimental values remains limited despite the inherent variability associated with post-peak concrete behavior. Considering that n_{d2} governs the curvature of the softening branch and is therefore influenced by complex interactions between material properties, strain localization, and fracture development, the obtained level of accuracy is considered satisfactory. The results suggest that the combined influence of f_{c0} and L/D , as incorporated in Eq. 13, captures the primary mechanisms controlling the evolution of post-peak curvature. Moreover, the exclusion of D avoids unnecessary model complexity while maintaining a high level of predictive performance, thereby resulting in a statistically robust and practically applicable formulation.

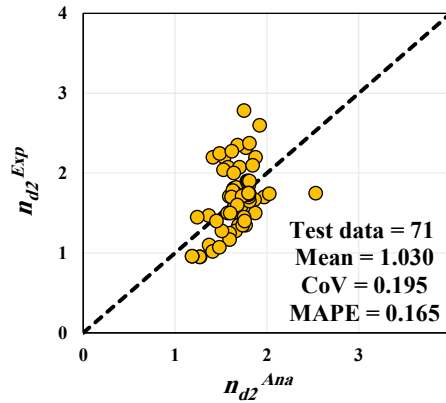


Fig. 8. Predictive performance of Eq. 13.

4. Validation of the proposed stress-strain relationship

To assess the predictive capability of the proposed stress-strain model, independent experimental results reported in the literature were selected for validation. The comparison focuses on the model's ability to reproduce the influence of the governing parameters identified during the calibration process, namely concrete compressive strength and specimen aspect ratio, on both the pre-peak and post-peak responses of unconfined concrete. Before presenting the validation results, the complete formulation of the proposed model is summarized in Table 3. It should be noted that each mathematical form was selected according to the characteristics of the response being represented. Power-law formulations were adopted for the empirical parameter relationships because they provide a compact and robust description of the nonlinear dependence of the model parameters on concrete strength and specimen geometry using only a few coefficients. The polynomial function was adopted for the ascending branch because it satisfies the required constitutive boundary conditions while allowing the curve shape to be governed by a single parameter. The logistic function was selected for the descending branch because it naturally represents the progressive strain-softening process through a smooth transition and a physically meaningful inflection point associated with the onset of damage localization.

Table 3. Summary of the proposed model.

Stress-strain relationship:

$$f_c = \left[\frac{1}{1 - n_{d1}} \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right)^{n_{d1}} - \frac{n_{d1}}{1 - n_{d1}} \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right) \right] f_{c0} \quad \text{for } \varepsilon_c \leq \varepsilon_{c0}$$

$$\frac{f_c}{f_{c0}} = \frac{1}{\left(\frac{\varepsilon_c - \varepsilon_{c0}}{\varepsilon_{ci0} - \varepsilon_{c0}} \right)^{n_{d2}} + 1} \quad \text{for } \varepsilon_c \geq \varepsilon_{c0}$$

in which:

$$\varepsilon_{c0} = 0.0011 \left(\frac{f_{c0} D}{L} \right)^{0.25}; \quad \frac{\varepsilon_{ci0}}{\varepsilon_{c0}} = 23.4 f_{c0}^{-0.59} \left(\frac{L}{D} \right)^{-0.56} \left(\frac{D}{150} \right)^{-0.1}; \quad n_{d1} = \frac{0.77 f_{c0}^{0.4} \left(\frac{L}{D} \right)^{0.1}}{1 + \frac{D}{300}}; \quad n_{d2} = 4.2 f_{c0}^{-0.17} \left(\frac{L}{D} \right)^{-0.36}$$

Fig. 9a compares the analytical predictions with the experimental stress-strain curves reported by Wischers [35] for unconfined concrete specimens with different concrete strength levels. As shown, the proposed model accurately captures the progressive change in stress-strain behavior associated with increasing compressive strength. In particular, the model successfully reproduces the increase in peak stress, the corresponding variation in peak strain, and the changes in the post-peak softening response. The agreement between analytical and experimental results indicates that the proposed formulations are capable of appropriately representing the influence of material strength on both the ascending and descending branches of the stress-strain curve. Moreover, the model correctly predicts the tendency of higher-strength concrete to exhibit a more localized and less ductile post-peak response compared with lower-strength concrete. The influence of specimen geometry is examined in Fig. 9b using the experimental results reported by Jansen Daniel and Shah Surendra [29]. The considered specimens possess similar material properties but different aspect ratios (L/D). As demonstrated, the proposed model adequately reproduces the observed effect of specimen slenderness on the stress-

strain response, particularly in the post-peak region where geometric effects become increasingly significant. The analytical curves correctly capture the reduction in deformation capacity and the modification of the softening branch associated with increasing aspect ratio. This behavior is consistent with the underlying assumption that post-peak deformation is governed by strain localization within a fracture zone whose relative contribution decreases as the specimen length increases. Overall, the comparisons presented in Fig. 9 demonstrate that the proposed model provides a realistic representation of the complete stress-strain response of unconfined concrete over a wide range of material strengths and geometric configurations. The ability of the model to accurately reproduce both the peak-stage characteristics and the post-peak softening behavior confirms the robustness of the proposed formulations and supports their applicability for analytical and numerical modeling of concrete columns.

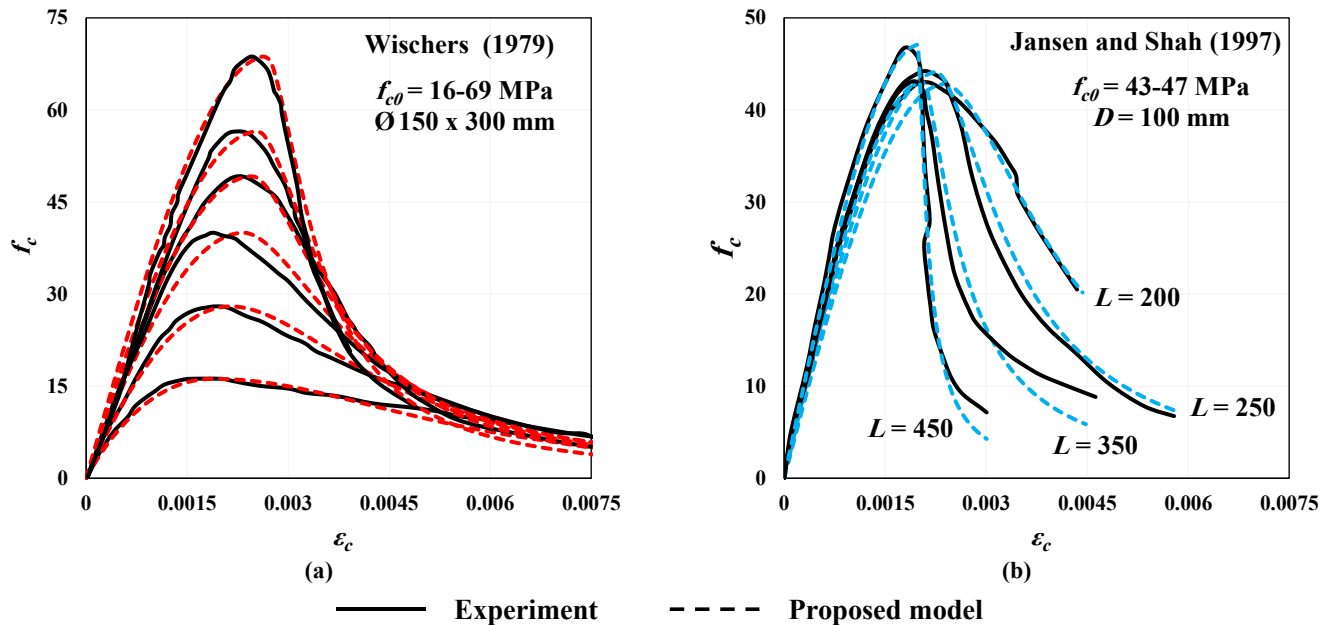


Fig. 9. Comparison of the results obtained from the experiments and the proposed model: (a) effect of concrete compressive strength; and (b) effect of specimen aspect ratio.

To rigorously demonstrate the global reliability, versatility, and predictive stability of the proposed stress-strain model, the analytical response curves are validated against a comprehensive set of independent experimental results, as compiled in Fig. 10. The experimental stress-strain data used for this validation were collected from thirteen independent historical research programs, namely those conducted by Ahmad and Shah [31], Dahl [28], Taerwe [36], Jansen Daniel and Shah Surendra [29], Nakamura and Higai [34], Watanabe et al. [23], Eid et al. [15], Desnerck et al. [37], Osorio et al. [32], Lim and Ozbakkaloglu [8], Hany Najwa et al. [38], Paultre et al. [26], and Gholampour et al. [20]. This extensive database ensures that the validation covers a wide spectrum of testing conditions and material characteristics.

As shown across all subfigures in Fig. 10, the proposed analytical model exhibits strong predictive capability by simultaneously capturing both the nonlinear ascending pre-peak response and the highly nonlinear post-peak softening behavior across all considered experimental datasets. The validation matrix spans a broad range of unconfined concrete column configurations, including varying geometric aspect ratios (L/D), cross-sectional dimensions (D), and concrete compressive strengths ranging from normal-strength concrete (17 MPa) to high-strength concrete (107 MPa). For high-strength specimens (e.g., $f_{c0} = 90, 94$, and 107 MPa), the model accurately reproduces the characteristic brittle response, including a nearly linear pre-peak branch followed by a rapid and pronounced post-peak stress degradation. This behavior is consistent with the reduced deformation capacity and enhanced strain localization typically observed in high-strength cementitious materials, which is effectively reflected by the proposed formulation. In contrast, for normal- and low-strength concrete specimens (e.g., $f_{c0} = 17$ and 20 MPa), the analytical model accurately captures the more gradual transition from peak stress to residual capacity, characterized by a smoother softening response and extended post-peak deformation range. This reflects the more distributed cracking process and higher energy dissipation capacity associated with lower-strength concrete matrices. Importantly, the model reproduces these contrasting behavioral regimes in a continuous and unified manner, without requiring case-specific recalibration of parameters. Furthermore, the validation confirms that the model remains robust across a wide range of specimen geometries, including cylinder diameters from $\varnothing 63$ mm to $\varnothing 200$ mm and specimen lengths up to 600 mm. The consistent agreement between analytical predictions and experimental observations across these diverse configurations demonstrates that the embedded size-dependent peak formulations, together with the calibrated shape parameters (ϵ_{c0} , ϵ_{ci0} , n_{d1} and n_{d2}), effectively capture both global scaling effects and localized macro-cracking mechanisms governing post-peak behavior. Overall, the close correspondence observed throughout Fig. 10 confirms that the proposed model provides a unified and physically consistent representation of the full compressive response of unconfined concrete. The demonstrated accuracy across a broad range of material strengths, geometric properties, and experimental programs further establishes its reliability as a general-purpose constitutive tool, applicable without the need for case-specific recalibration.

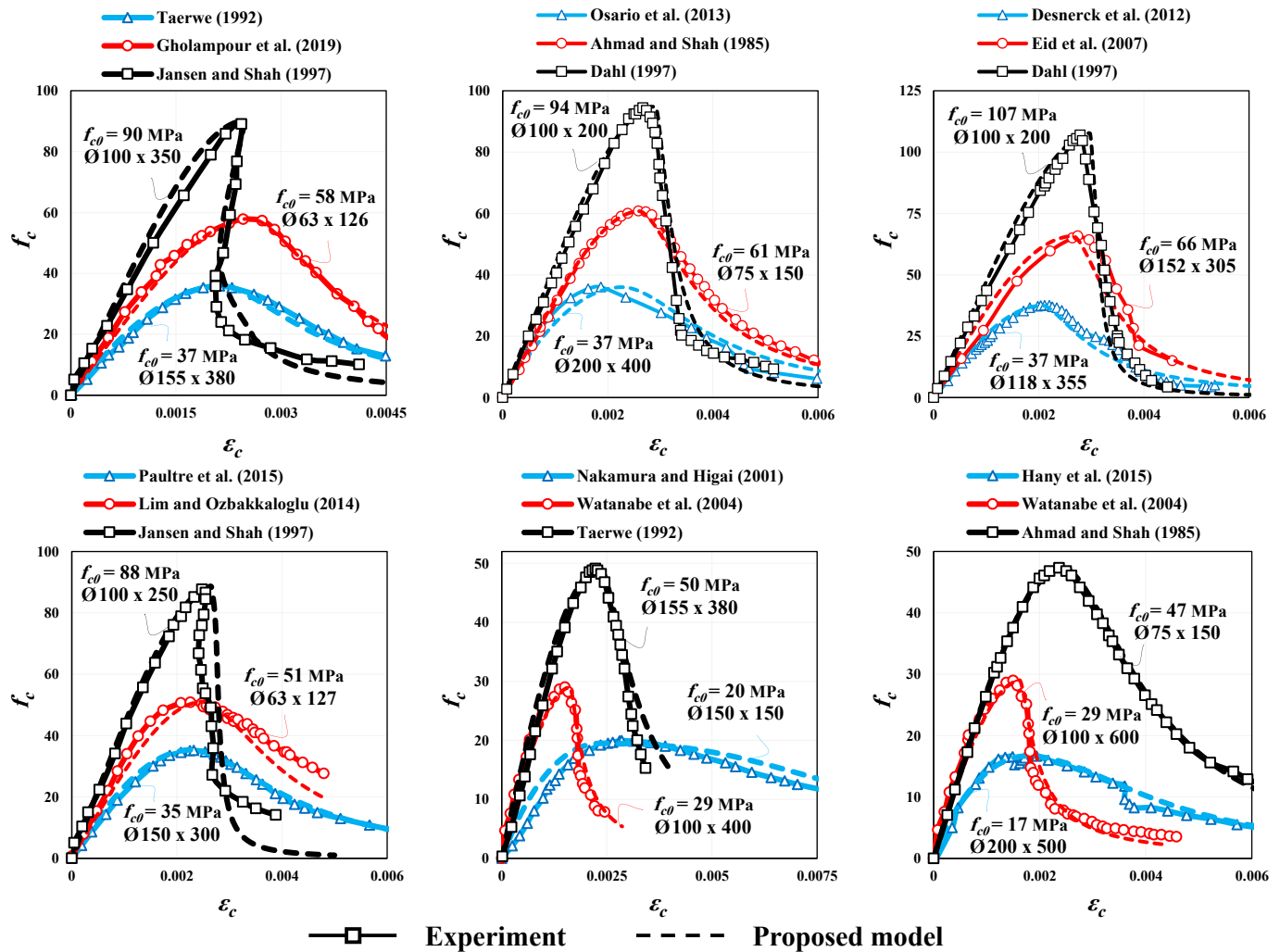


Fig. 10. Comparison between results obtained from the experiments and the proposed model.

5. Conclusions

This study proposed a unified size-dependent stress-strain model for unconfined concrete under monotonic axial compression. The formulation was established through regression analysis of a comprehensive experimental database and validated against independent experimental stress-strain curves covering a broad range of concrete strengths and specimen geometries. By integrating simplified empirical relationships with physically based constitutive functions, the proposed model provides a practical representation of both the ascending and descending branches of the compressive response. Based on the results obtained, the main findings of this study are summarized as follows:

- A unified constitutive formulation was developed that explicitly incorporates the effects of concrete strength, specimen size, and aspect ratio, enabling consistent prediction of the compressive response of both normal- and high-strength unconfined concrete.
- New regression-based expressions were established for the peak strain and the governing pre-peak and post-peak parameters, providing a simple yet robust representation of their dependence on material strength and specimen geometry.
- Introducing a variable pre-peak shape parameter allows the model to capture experimentally observed changes in the normalized ascending response that cannot be represented by conventional formulations employing a fixed second-degree parabola.
- The proposed two-branch formulation accurately reproduces both the ascending and descending branches of the compressive stress-strain response while maintaining mathematical simplicity and computational efficiency.
- Validation against independent experimental stress-strain curves demonstrated that the proposed model reliably predicts the complete compressive response over a broad range of concrete strengths and specimen geometries.

Overall, the results demonstrate that the proposed stress-strain model provides a physically consistent and computationally efficient representation of the complete compressive response of unconfined concrete. By explicitly incorporating size effects and fracture localization into a compact analytical framework, the model offers a practical constitutive relationship for nonlinear structural analysis and performance-based assessment of reinforced concrete structures (e.g., Shayanfar and Akbarzadeh Bengar [39-41], Shayanfar et al. [10, 42-44], and Dashtaki et al. [45]). In particular, the proposed formulation can be readily implemented in nonlinear finite element models and other computational analysis frameworks to improve the simulation of concrete members

with different strength classes and geometric configurations (e.g., Nematzadeh et al. [46], Shayanfar et al. [47], Tariq et al. [48], and Khant et al. [49]), thereby supporting more reliable structural assessment and design.

It should be noted that the proposed formulation is applicable to the monotonic compressive response of unconfined concrete. Extension to cyclic loading is beyond the scope of the present study and would require coupling the proposed monotonic envelope with appropriate unloading-reloading rules and a damage evolution law capable of representing stiffness degradation, strength deterioration, and irreversible strain accumulation under repeated loading. This represents a promising direction for future research.

Statements & declarations

Author contributions

Javad Shayanfar: Conceptualization, Methodology, Data curation, Software, Validation, Writing - Original Draft.

Joaquim A. O. Barros: Conceptualization, Methodology, Project administration, Resources, Writing - Review & Editing.

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Data availability

All data are available in the manuscript.

Declarations

The authors declare no conflict of interest.

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